

# Excess of loss reinsurance with reinstatements: premium calculation and ruin probability of the cedent

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## 1. Introduction

In a previous paper, Walhin and Paris (2000) studied the effect of reinstatements on the probability of ruin of the cedent. The theoretical results have been deduced under the hypothesis that the premium was calculated according to the expected value principle and the study allowed to evaluate the adjustment coefficient and the expected gain of different reinsurance agreements.

In this paper we analyse the effect of a premium calculated with the standard deviation principle and with the PH transform principle. The adjustment coefficient is calculated as well as finite time ruin probabilities.

The tools described in this paper give the decision maker the opportunity to choose objectively between different reinsurance treaties.

## 2. Notations

As usual we will assume through this paper that the claim amount distributions are arithmetic distributions. If it is not the case it is also possible to transform any distribution into an arithmetic distribution (see for example Walhin and Paris (1998) or Gerber (1982)).

Moreover we assume that the number of claims in a year is in the  $(a, b, 0)$  class in order to be able to use typical Panjer's algorithms.

We will use the following notations. Some of them are inspired from Sundt (1991).

- the number of claims is a random variable  $N$
- the severity of claims is a random variable  $X$
- the  $X_i$ 's are iid and independent of  $N$
- $S = X_1 + \dots + X_N$  denotes the aggregate claims of the portfolio for the period in consideration
- $D$  is the deductible of the excess of loss reinsurance. It is also the retention of the Cedent
- $L$  is the maximum liability of the reinsurer per claim
- $R_i = \min(L, \max(X_i - D, 0))$  is the part of claim  $i$  for which the Reinsurer is liable
- $A_i = X_i - R_i$  is the part of claim  $i$  for which the Cedent is liable (retained part of the risk)
- $S_R = R_1 + \dots + R_N$  is the aggregate part of the reinsurer if there is an unlimited number of free reinstatements
- $S_A = A_1 + \dots + A_N$  is the aggregate part of the insurer if there is an unlimited number of free reinstatements
- $p_0$  is the initial (deterministic) reinsurance premium
- $k$  is the number of reinstatements
- $c_i$  is the price of the  $i$ th reinstatement
- $p = p_0/L$  is the rate on line

- If there are reinstatements the aggregate parts of the Cedent and Reinsurer become

$$S_{\text{Ced}} = S_A + \max(0, S_R - (k+1)L) + \frac{p_0}{L} \sum_{i=1}^k c_i \min(L, \max(0, S_R - (i-1)L)) \quad (1)$$

$$S_{\text{Re}} = \min((1+k)L, S_R) - \frac{p_0}{L} \sum_{i=1}^k c_i \min(L, \max(0, S_R - (i-1)L)) \quad (2)$$

$S_{\text{Ced}}$  is the total claims of the Ceding Company. We assume that the reinstatement premiums are considered as claims for the Ceding Company.

$S_{\text{Re}}$  is the total claims of the Reinsurer. We consider that the reinstatement premiums the Reinsurer will receive let decrease his total claims.

- $r_i = \min(\max(0, S_R - iL), L)$  is the coverage of the  $i$ th reinstatement
- $c_i p_0 (r_{i-1}/L)$  is the  $i$ th reinstatement premium (pro rata capita)
- $R_k = \sum_{i=0}^k r_i = \min(S_R, (k+1)L)$  is the aggregate claim payments of the Reinsurer
- $T = p_0 \left( 1 + \frac{1}{L} \sum_{i=1}^k c_i r_{i-1} \right)$  is the total reinsurance premium. It is a random variable
- $d_i = \mathbb{E} r_i$
- $D_k = \mathbb{E} R_k$
- $v_{ij} = \text{Cov}(r_i, r_j)$
- $v_i = \text{Var} r_i$
- $V_k = \text{Var} R_k$
- $\gamma$ : security loading with the standard deviation premium principle
- $\rho$ : risk aversion index for PH transform premium principle
- $\alpha$ : safety loading with the expected value premium principle

### 3. The standard deviation premium principle

This case has been studied in Sundt (1991). The initial reinsurance premium  $p_0$  is obtained from the equation

$$\mathbb{E}T = \mathbb{E}R_k + \gamma \sqrt{\text{Var}(R_k - T)} \quad (3)$$

Introducing some further notations:

$$A = L + \sum_{i=1}^k c_i d_{i-1}$$

$$B = \text{Var} \left( \sum_{i=1}^k c_i r_{i-1} \right)$$

$$C = \text{Cov} \left( \sum_{i=1}^k c_i r_{i-1}, R_k \right)$$

The rate on line  $p$  is deduced from the equation

$$pA - D_k = \gamma \sqrt{V_k + p^2 B - 2pC} \quad (4)$$

Sundt (1991) claims that an acceptable solution will be provided at least if  $\gamma < \frac{A}{\sqrt{B}}$ . We show now the exact condition on  $\gamma$ .  
(4) leads to

$$p^2(A^2 - \gamma^2 B) - 2(AD_k - C\gamma^2)p + D_k^2 - \gamma^2 V_k = 0 \quad (5)$$

The discriminant will be positive if

$$\gamma < \sqrt{\frac{A^2 V_k + BD_k^2 - 2CAD_k}{BV_k - C^2}}$$

Obviously this bound on  $\gamma$  is better than Sundt's bound:

$$\begin{aligned} \frac{A^2}{B} &< \frac{A^2 V_k + BD_k^2 - 2CAD_k}{BV_k - C^2} \\ &\Leftrightarrow \\ A^2 BV_k - A^2 C^2 &< A^2 BV_k + B^2 D_k^2 - 2ABCD_k \\ &\Leftrightarrow \\ 0 &< A^2 \left( C + B \frac{D_k}{A} \right)^2 \end{aligned}$$

which is always true.

The rate on line  $p$  is then given by the largest solution of (5).

Note that (3) can be written as

$$p_0 = \mathbb{E}S_{Re} + \gamma \sqrt{\text{Var}(S_{Re})} \quad (6)$$

with  $S_{Re}$  a function of  $p_0$ . So  $p_0$  can also be found by an iterative process through (6).

Note that if one uses the algorithm

$$p_0^{(i)} = f(p_0^{(i-1)})$$

with  $f = \mathbb{E}S_{Re} + \gamma \sqrt{\text{Var}(S_{Re})}$ ,  $f$  should satisfy the Lipschitz condition in order to get a fixed point. In practice this is not always the case and divergence may occur. Then another iterative algorithm has to be used like the bisection method for example.

Nevertheless, the solution, if it exists is always available explicitly for the standard deviation principle.

#### 4. The PH transform premium principle

Wang (1996) introduced several risk adjusted premium calculation principles.

These principles have been studied in Silva and Centeno (1998) where they conclude that only the proportional hazard (PH) transform premium principle has a different behaviour than the expected value premium principle.

In this section we will concentrate on the evaluation of the premium of an excess of loss treaty with reinstatements with the PH transform premium principle.

For a risk  $S$ , the premium calculated according to the PH transform premium principle is given by

$$P = \int_0^{\infty} (1 - F_S(t))^{1/\varrho} dt, \quad \varrho \geq 1$$

where  $\varrho$  is called the risk aversion index.

In our case, we are interested in

$$p_0 = \int_0^{\infty} (1 - F_{S_{Re}}(t))^{1/\varrho} dt$$

where  $S_{Re}$  is itself a function of  $p_0$ . The initial reinsurance premium  $p_0$  will be given by iterations.

The remark of the preceding section about divergence is still valid.

Note that, if  $c_i(p_0/L) > 1$ , a case that would not happen in practice, the distribution of  $S_{Re}$  is not on the positive numbers and so the PH transform has to be extended as

$$p_0 = \int_{-\infty}^0 [(1 - F_{S_{Re}}(t))^{1/\varrho} - 1] dt + \int_0^{\infty} (1 - F_{S_{Re}}(t))^{1/\varrho} dt$$

## 5. The aggregate claims distribution of the ceding company

We saw in section 2 that the distribution of  $S_{Ced}$  depends on  $S_A$  and  $S_R$  that are dependent. Walhin and Paris (2000) have used an algorithm that authorizes to find the joint distribution of  $(S_A, S_R)$  such that it is then possible to find the distribution of  $S_{Ced}$ . The bivariate Panjer's algorithm follows:

### Proposition

Let

$$(S_A, S_R) = \left( \sum_{i=1}^N A_i, \sum_{i=1}^N R_i \right)$$

with  $(A_i, R_i)$  iid, arithmetic and independent of  $N$   
 $N$  belonging to Panjer's class  $((a, b, 0)$  class), i.e. such that

$$\frac{p(n)}{p(n-1)} = a + \frac{b}{n}, \quad n > 0.$$

Then, if  $\psi_N(z)$  denotes the probability generating function of  $N$ , we have

$$f_{(S_A, S_R)}(0, 0) = \psi_N(f_{(A, R)}(0, 0)) \quad (7)$$

$$f_{(S_A, S_R)}(j, k) = \frac{1}{1 - a f_{(A, R)}(0, 0)} \sum_m^j \sum_n^k \left[ a + b \frac{m}{j} \right] \cdot f_{(S_A, S_R)}(j-m, k-n) f_{(A, R)}(m, n), \quad j \geq 1 \quad (8)$$

$$f_{(S_A, S_R)}(j, k) = \frac{1}{1 - a f_{(A, R)}(0, 0)} \sum_m^j \sum_n^k \left[ a + b \frac{n}{k} \right] \cdot f_{(S_A, S_R)}(j-m, k-n) f_{(A, R)}(m, n), \quad k \geq 1 \quad (9)$$

where we use the notation

$$\sum_m^j \sum_n^k f(m, n) = \sum_{m=0}^j \sum_{n=0}^k f(m, n) - f(0, 0).$$

The distribution of  $S_{\text{Ced}}$  follows immediately from (1).

Knowing the distribution of  $S_{\text{Ced}}$  we will be able to compute the adjustment coefficient as well as finite time ruin probabilities.

## 6. Finite and discrete time ruin probabilities with non integer premium

Let  $S_i$  the aggregate claims distribution of a portfolio for year  $i$ . The premium corresponding to this risk is  $P > \mathbb{E} S$ .

The  $S_i$ 's,  $1 \leq i \leq n$  are iid and distributed with cdf  $F$  and dff.

Assuming an initial surplus  $u$ , the surplus at time  $t$  is given by

$$U(t) = u + Pt - \sum_{i=1}^t S_i.$$

The finite and discrete time probability of ruin in the interval  $[0, n]$  is given by

$$\psi(u, n) = \mathbb{P}(\exists t \leq n | U(t) < 0).$$

De Vylder and Goovaerts (1988) gave the following recursive algorithm for the computation of  $\psi(u, n)$ :

$$\begin{aligned} \psi(u, 1) &= 1 - F(u + P) \\ \psi(u, n) &= 1 - F(u + P) + \int_0^{u+P} \psi(u + P - y, n - 1) dF(y) \end{aligned}$$

This algorithm is of practical interest when  $S$  is arithmetic together with  $u$  and  $P$  integers. Then it becomes

$$\begin{aligned} \psi(u, 1) &= 1 - F(u + P) \\ \psi(u, n) &= 1 - F(u + P) + \sum_{j=0}^{u+P} \psi(u + P - j, n - 1) f(j) \end{aligned}$$

Clearly for our case we will not have an arithmetic  $S$  neither an integer premium.

Let us analyse how to adapt the algorithm if  $P$  is non integer when  $S$  is arithmetic.

Dickson and Waters (1996) claim, without proof, that the algorithm becomes

$$\psi(u, 1) = 1 - F(u + \underline{P}) \tag{10}$$

$$\psi(u, n) = 1 - F(u + \underline{P}) + \int_0^{u+\underline{P}} \psi(u + \bar{P} - y, n - 1) dF(y) \tag{11}$$

where  $\underline{P}$  denotes the greatest integer contained in  $P$  and  $\bar{P}$  denotes the least integer greater than  $P$ . Let us use a numerical example to show that this conjecture is not true.

Let  $S$  be distributed as

Table 1. Non integer premium

|                    |           |     |     |     |
|--------------------|-----------|-----|-----|-----|
| $S$                | 1         | 2   | 3   | 4   |
| $f_S(x)$           | 0.5       | 0.2 | 0.1 | 0.2 |
| $\mathbb{E} S = 2$ | $P = 2.4$ |     |     |     |

Using formulae (10) and (11) we find

Table 2. Ruin probabilities

|                       |
|-----------------------|
| $\psi(0, 4) = 0.37$   |
| $\psi(1, 4) = 0.2366$ |
| $\psi(2, 4) = 0.0524$ |

Now let us rescale  $S$  in order to have an integer premium:

Table 3. Integer premium; rescaled distribution

|                     |          |     |     |     |
|---------------------|----------|-----|-----|-----|
| $S'$                | 10       | 20  | 30  | 40  |
| $f_{S'}(x)$         | 0.5      | 0.2 | 0.1 | 0.2 |
| $\mathbb{E}S' = 20$ | $P = 24$ |     |     |     |

We find

Table 4. Ruin probabilities; rescaled distribution

|                         |
|-------------------------|
| $\psi'(0, 4) = 0.5103$  |
| $\psi'(10, 4) = 0.327$  |
| $\psi'(20, 4) = 0.1291$ |

$\psi(u, n)$  and  $\psi'(10u, n)$  should agree. It is not the case. The formula of Dickson and Waters (1996) is not general.

Of course rescaling the distribution of  $S$  is not efficient numerically. As  $\psi(u, 1)$  is always available, a solution may consist in evaluating  $\psi(u, n)$ ,  $n \geq 2$  by linear interpolation where  $\psi(u + P - j, n - 1)$  is approximated by

$$(P - \underline{P}) \psi(u + \bar{P} - j, n - 1) - (P - \bar{P}) \psi(u + \underline{P} - j, n - 1).$$

Using this approximation we find

Table 5. Approximate ruin probabilities

|                               |
|-------------------------------|
| $\psi^{app}(0, 4) = 0.487288$ |
| $\psi^{app}(1, 4) = 0.322453$ |
| $\psi^{app}(2, 4) = 0.131086$ |

We note that this approximate solution is not quite good.

The exact solution may be calculated at a cost that is however lower than the cost for rescaling.

Let  $w = \max(j | f(j) > 0)$ .

In order to find  $\psi(u, n)$ , apply the algorithm

```

For t = 1 to n - 1
  For i = 0 to min(w, u + (n - t) P)
    Evaluate  $\psi(u + (n - t) P - i, t)$ 
  Next i
Next t

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## 7. Finite and discrete time ruin probabilities

We are now interested in calculating finite time ruin probabilities when excess of loss reinsurance with reinstatements is bought by the ceding company.

It is clear that the retained premium by the Cedent ( $P - p_0$ ) will not be an integer. Moreover, the distribution of  $S_{\text{Ced}}$  is not arithmetic because of the paid reinstatements.

Following the lines of section 6, a general algorithm might be constructed in order to evaluate finite time ruin probabilities without rescaling the distribution of  $S_{\text{Ced}}$ . However this would be cumbersome and we will only calculate the finite time ruin probabilities by approximations.

Let  $S_{\text{Ced}}^+$  and  $S_{\text{Ced}}^-$  be arithmetic random variables defined such that

$$\begin{aligned} F_{S_{\text{Ced}}^+}(s) &\leq F_{S_{\text{Ced}}}(s) \quad \forall s \geq 0, \\ F_{S_{\text{Ced}}^-}(s) &\geq F_{S_{\text{Ced}}}(s) \quad \forall s \geq 0. \end{aligned}$$

It is clear from the definition of  $\psi(u, n)$  that we will have

$$\psi^-(u, n) \leq \psi(u, n) \leq \psi^+(u, n).$$

So we will be able to give an upper bound and a lower bound on the probability of ruin by using the exact procedure described in section 6.

An approximate procedure is simply given by using linear interpolation where  $\psi(u + P - j, n - 1)$  is approximated by

$$(u + P - j - (\underline{u + P - j})) \psi(\overline{u + P - j}, n - 1) - (u + P - j - (\overline{u + P - j})) \psi(\underline{u + P - j}, n - 1)$$

with  $j$  non necessary integer.

## 8. Numerical examples

Let us use the same numerical example than in Walhin and Paris (2000).

Let  $X$  be the claim amount distribution

Table 6. Claim amount distribution

|          |     |      |      |     |      |      |      |      |      |      |
|----------|-----|------|------|-----|------|------|------|------|------|------|
| $X$      | 1   | 2    | 3    | 4   | 5    | 6    | 8    | 10   | 12   | 14   |
| $f_X(x)$ | 0.2 | 0.15 | 0.15 | 0.2 | 0.06 | 0.06 | 0.06 | 0.05 | 0.04 | 0.03 |

The number of claims is Poisson distributed with mean  $\lambda = 3$ .

The reinsurance layer is  $4 \times 6$ .

Let us assume that the Ceding company charges a premium with a loading of 50% according to the expected premium principle.

**Case 1** Reinsurance premium calculated according to the expected value premium principle with loading  $\alpha = 100\%$

The reinsurance premium is calculated according to the net premium principle. The pure premium obtained is then loaded with  $1 + \alpha$ .

Table 7. Reinsurance premium

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 2.9184 | 3.5101 | 3.5910 | 3.5993 |
| 50%  |        | 2.9686 | 2.9450 | 2.9395 |
| 100% |        | 2.5713 | 2.4959 | 2.4842 |
| 150% |        | 2.2687 | 2.1657 | 2.1510 |

Table 8. Adjustment coefficient

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 0.1019 | 0.1142 | 0.1223 | 0.1252 |
| 50%  |        | 0.1064 | 0.1070 | 0.1065 |
| 100% |        | 0.1008 | 0.0972 | 0.0953 |
| 150% |        | 0.0965 | 0.0906 | 0.0880 |
| EG   | 4.9758 | 4.6799 | 4.6395 | 4.6353 |

Table 9. Expected gain

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 4.9758 | 4.6799 | 4.6395 | 4.6353 |
| 50%  |        | 4.6799 | 4.6395 | 4.6353 |
| 100% |        | 4.6799 | 4.6395 | 4.6353 |
| 150% |        | 4.6799 | 4.6395 | 4.6353 |

As we saw in Walhin and Paris (2000), the expected gain is independent of the price of reinstatements and the adjustment coefficient diminishes when the price of reinstatements increases.

**Case 2** Reinsurance premium calculated according to the PH transform premium principle with risk aversion index  $\rho = 2$

Table 10. Reinsurance premium

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 2.4097 | 3.4882 | 3.8841 | 4.0097 |
| 50%  |        | 2.6807 | 2.7047 | 2.6992 |
| 100% |        | 2.1768 | 2.0748 | 2.0343 |
| 150% |        | 1.8324 | 1.6828 | 1.6323 |

Table 11. Adjustment coefficient

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 0.1088 | 0.1146 | 0.1167 | 0.1167 |
| 50%  |        | 0.1127 | 0.1133 | 0.1131 |
| 100% |        | 0.1113 | 0.1111 | 0.1107 |
| 150% |        | 0.1103 | 0.1096 | 0.1091 |

Table 12. Expected gain

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 5.4846 | 4.7019 | 4.3464 | 4.2249 |
| 50%  |        | 5.0204 | 4.9324 | 4.9296 |
| 100% |        | 5.2191 | 5.2454 | 5.2872 |
| 150% |        | 5.3550 | 5.4401 | 5.5034 |

We see on this example that the reinsurance premium decreases more rapidly with the price of reinstatement than with the expected value principle.

**Case 3** Reinsurance premium calculated according to the standard deviation principle with security loading  $\gamma = 0.8$

Table 13. Reinsurance premium

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 2.9098 | 3.6707 | 3.8148 | 3.8343 |
| 50%  |        | 2.7251 | 2.6209 | 2.5969 |
| 100% |        | 2.1770 | 1.9983 | 1.9635 |
| 150% |        | 1.8189 | 1.6160 | 1.5782 |

Table 14. Adjustment coefficient

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 0.1020 | 0.1116 | 0.1181 | 0.1204 |
| 50%  |        | 0.1117 | 0.1155 | 0.1159 |
| 100% |        | 0.1113 | 0.1136 | 0.1132 |
| 150% |        | 0.1107 | 0.1123 | 0.1114 |

Table 15. Expected gain

| c    | k = 0  | k = 1  | k = 2  | k = 3  |
|------|--------|--------|--------|--------|
| 0%   | 4.9844 | 4.5194 | 4.4157 | 4.4004 |
| 50%  |        | 4.9679 | 5.0346 | 5.0549 |
| 100% |        | 5.2189 | 5.3555 | 5.3897 |
| 150% |        | 5.3759 | 5.5509 | 5.5930 |

We can make the same kind of remarks that for case 2.

Unfortunately it is quite complicated to draw conclusions about the comparison of the behaviour of these premium principles.

#### Case 4 High loadings

In case of high loadings, we observe abnormal behaviours of the initial reinsurance premiums. Let us assume a first cover with two layers: 4 xs 6 and 4 xs 10. Another cover might be

8 xs 6. It is clear that if the number of reinstatements is the same for every layers, the cover 2 provides a better protection for the Ceding Company and should therefore have an initial higher cost.

Let us assume that  $\varrho = 5$  and  $\gamma = 1.5$ . We find

Table 16. Reinsurance premiums with high loading

|                | 4 xs 6 1 reinst@100% | 4 xs 10 1 reinst@100% | 8 xs 6 1 reinst@100% |
|----------------|----------------------|-----------------------|----------------------|
| $\varrho = 5$  | 3.09                 | 2.51                  | 4.81                 |
| $\gamma = 1.5$ | 2.73                 | 1.67                  | 4.33                 |

We remark on this example that the cover with one large layer is less expensive than the combination of two short layers. This is not logical and one should therefore be careful when using high loadings.

### Finite time ruin probabilities

We now compare the finite time ruin probabilities for two cases:

Table 17. Lower bound

|          | PH transform<br>1 reinstatement@100% |         | PH transform<br>3 reinstatements@150% |         |
|----------|--------------------------------------|---------|---------------------------------------|---------|
|          | $t = 3$                              | $t = 5$ | $t = 3$                               | $t = 5$ |
|          | EG = 5.2191; r = 0.1113              |         | EG = 5.5034; r = 0.1091               |         |
| $u = 0$  | 0.2248                               | 0.2343  | 0.2372                                | 0.2484  |
| $u = 10$ | 0.0562                               | 0.0623  | 0.0667                                | 0.0742  |
| $u = 20$ | 0.0123                               | 0.0150  | 0.0157                                | 0.0191  |
| $u = 30$ | 0.0025                               | 0.0034  | 0.0032                                | 0.0044  |
| $u = 40$ | 0.0005                               | 0.0008  | 0.0006                                | 0.0010  |

Table 18. Upper bound

|          | PH transform<br>1 reinstatement@100% |         | PH transform<br>3 reinstatements@150% |         |
|----------|--------------------------------------|---------|---------------------------------------|---------|
|          | $t = 3$                              | $t = 5$ | $t = 3$                               | $t = 5$ |
|          | EG = 5.2191; r = 0.1113              |         | EG = 5.5034; r = 0.1091               |         |
| $u = 0$  | 0.2554                               | 0.2677  | 0.2674                                | 0.2816  |
| $u = 10$ | 0.0693                               | 0.0780  | 0.0810                                | 0.0913  |
| $u = 20$ | 0.0161                               | 0.0201  | 0.0204                                | 0.0255  |
| $u = 30$ | 0.0035                               | 0.0049  | 0.0044                                | 0.0064  |
| $u = 40$ | 0.0007                               | 0.0012  | 0.0009                                | 0.0015  |

Table 19. Approximate expression

|        | PH transform<br>1 reinstatement@ 100% |        | PH transform<br>3 reinstatements@ 150% |        |
|--------|---------------------------------------|--------|--|--------|
|        | t = 3                                 | t = 5  | t = 3                                  | t = 5  |
|        | EG = 5.2191; r = 0.1113               |        | EG = 5.5034; r = 0.1091                |        |
| u = 0  | 0.2226                                | 0.2329 | 0.2560                                 | 0.2692 |
| u = 10 | 0.0553                                | 0.0619 | 0.0753                                 | 0.0845 |
| u = 20 | 0.0120                                | 0.0149 | 0.0181                                 | 0.0226 |
| u = 30 | 0.0025                                | 0.0035 | 0.0037                                 | 0.0054 |
| u = 40 | 0.0005                                | 0.0008 | 0.0007                                 | 0.0012 |

The results confirm the intuition given by the adjustment coefficient. The lower the adjustment coefficient the larger the probability of ruin.

The advantage of calculating the probability of ruin is that it is a quantity more speaking for the decision maker.

## 9. Conclusion

We have reviewed in this paper some premium principles applied to the calculation of the reinsurance premium of an excess of loss agreement with reinstatements.

All the concepts can be extended to other types of reinsurance clauses: sliding premium, aggregate deductible, ...

The tools described in this paper are important in order to compare two reinsurance agreements from the point of view of expected gain and risk. The risk is studied by calculating the adjustment coefficient or finite discrete time ruin probabilities.

Obviously, as usual, a reinsurance premium has to be loaded for commission, administration costs and profit. Nevertheless, the procedures given in this paper can be used when commercial reinsurance premiums are given for different agreements. It is then possible to give the decision maker the tools to decide which agreement he has to choose.

## REFERENCES

- De Vylder, F., M. J. Goovaerts* (1988): Recursive Calculation of Finite-Time Ruin Probabilities. *Insurance: Mathematics and Economics*, 7: 1–7.
- Dickson, D. C. M., H. R. Waters* (1966): Reinsurance and Ruin. *Insurance: Mathematics and Economics*, 19: 61–80.
- Gerber, H. U.* (1982): On the Numerical Evaluation of the Distribution of Aggregate Claims and its Stop-Loss Premiums. *Insurance: Mathematics and Economics*, 1: 13–18.
- Panjer, H. H.* (1981): Recursive Evaluation of a Family of Compound Distributions. *Astin Bulletin*, 12: 22–26.
- Silva, J. M., M. Centeno* (1998): Comparing Risk Adjusted Premiums from the Reinsurance Point of View. *Astin Bulletin*, 28: 221–239.
- Sundt, B.* (1991): On Excess of Loss Reinsurance with Reinstatement. *Bulletin of the Swiss Actuaries*, 1991: 1–15.
- Walhin, J. F., J. Paris* (1998): On the Use of Equispaced Discrete Distributions. *Astin Bulletin*, 28: 241–255.

- Walhin, J. F., J. Paris (2000): Excess-of-Loss Reinsurance with Reinstatements and Ruin of the Cedent. *Blätter Deutsche Gesellschaft für Versicherungsmathematik*, 24: 615–627.
- Wang, S. (1996): Premium Calculation by Transforming the Layer Premium Density. *Astin Bulletin*, 26: 71–92.

### *Zusammenfassung*

Schadenexzedenten-Rückversicherung mit Wiederauffüllung:  
Berechnung der Prämie und Ruinwahrscheinlichkeit des Zedenten

Die Prämien für Schadenexzedentenverträge mit Wiederauffüllung werden auf Basis des Standardabweichung-Prämienprinzips und der PH-Transformation kalkuliert. Die praktische Anwendung dieser Prämienprinzipien wird besprochen.

Der bivariate Panjers Algorithmus wird benutzt, um die zeitendliche Ruinwahrscheinlichkeit des Zedenten zu schätzen, wenn dieser Schadenexzedentenverträge mit Wiederauffüllung kauft.

### *Summary*

Excess of loss reinsurance with reinstatements:  
premium calculation and ruin probability of the cedent

Premiums for excess of loss treaties with reinstatements are calculated according to the standard deviation and PH transform premium principles. Some comments are given regarding the practical use of these premium principles.

The bivariate Panjer's algorithm is used in order to find finite time ruin probabilities of the Ceding Company when it buys excess of loss treaties with reinstatements.

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