MULTI-EVENT BONUS-MALUS SCALES

SANDRA PITREBOIS[‡], MICHEL DENUIT[†*]
& JEAN-FRANÇOIS WALHIN[*‡]

[‡]Secura
Avenue des Nerviens, 9-31 boîte 6
B-1040 Bruxelles, Belgium

[†]Institut de Statistique
Université Catholique de Louvain
Voie du Roman Pays, 20
B-1348 Louvain-la-Neuve, Belgium

[*]Institut des Sciences Actuarielles
Université Catholique de Louvain
Grand-Rue, 54
B-1348 Louvain-la-Neuve, Belgium

First version: August 30, 2004

Revision: September 24, 2005

Abstract

This paper is devoted to the design of bonus-malus scales involving different types of claims. Typically, claims with or without bodily injuries, or claims with full or partial liability of the insured driver, are distinguished and entail different penalties. Under mild assumptions, claim severities can also be taken into account in this way. Numerical illustrations enhance the interest of the approach.

Key words and phrases: Bonus-Malus system, Markov chains, A priori risk classification, A posteriori ratemaking, Relativities

1 Introduction and Motivation

Many important factors cannot be taken into account *a priori* when pricing motor third party liability insurance products. For instance, swiftness of reflexes, aggressiveness behind the wheel or knowledge of the highway code cannot be integrated into risk classification. Consequently, tariff cells are still quite heterogeneous despite the use of many classification variables. This residual heterogeneity typically causes overdispersion: data involving claim counts exhibit variability exceeding that explained by Poisson models. This can be modelled by a random effect in a statistical model.

It is reasonable to believe that the hidden characteristics are partly revealed by the number of claims reported by the policyholders. Hence the adjustment of the premium on the basis of the individual claims experience in order to restore fairness among policyholders. In that respect, the allowance of past claims in a rating model derives from an exogeneous explanation of serial correlation for longitudinal data. In this case, correlation is only apparent and results from the revelation of hidden features in the risk characteristics.

Once estimated, the statistical model incorporating the portfolio heterogeneity can be used to perform prediction on longitudinal data and allows for experience rating in motor insurance. In an empirical Bayesian setting, the prediction is derived from the expectation of a random effect with respect to a posterior distribution taking into account the history of the individual. This is the topic of credibility theory. We refer the interested reader to Chapter 7 of Kaas, Goovaerts, Dhaene & Denuit (2001) for more details, as well as to the papers by Dionne & Vanasse (1989, 1992) and Pinquet (2000).

In many countries, insurance companies integrate past claims histories in ratemaking with the help of bonus-malus systems. Such systems can be seen as commercial simplifications of credibility mechanisms. When a bonus-malus system is in force, the policyholders move inside a scale according to the number of claims they file (going up when claims are reported and down when they do not file any claim). The amount of premium is then obtained by multiplying the reference premium with a percentage attached to the level occupied by the policyholder in the scale. All policies in the same tariff class can then be partitioned according to the level they occupy in the bonus-malus scale. Therefore, bonus-malus scales refine the a priori risk classification.

As pointed out by Lemaire (1995), all bonus-malus systems in force throughout the world (with the exception of Korea) penalize the number of reported claims, without taking the cost of these claims into account. In Chapter 13 of his book, this author applied a model proposed by Picard (1976) to Belgian data. This credibility model allows to subdivide the claims into two categories, small and large losses. Instead of determining a limiting amount (such a criterion would lead to substantial practical problems, due to the time needed to evaluate the cost of the claim and endless arguments with policyholders who caused a claim slightly above the limit), Lemaire (1995) distinguished the accidents that caused property damage only from those that caused bodily injuries. Since the latter cost much more on average, this approach implicitly integrates the cost of the claim in a posteriori premium corrections. The credibility model proposed by Lemaire (1995) is based on a Poisson-Gamma mixture, and assumes that given the expected annual claim frequency of the policyholder, the frequency of claims with bodily injuries conforms to a Beta distribution. This approach can be extended to several categories of claims using a Dirichlet distribution (that is, a suitable multivariate Beta distribution).

In this paper, we work in the framework of bonus-malus scales (and not of credibility models) and we deal with several types of events (assuming a multinomial partitioning scheme, so avoiding to use multivariate Beta distributions). As mentioned above, all the classical bonus-malus systems are based on a single type of event: the occurrence of claims at fault, no matter of their severity or whether the policyholders are only partially liable for them. This over-simplification can be regarded as problematic for commercial purposes: it seems desirable to integrate the severity of the claims and to recognize the partial liability of the policyholder. For example, the bonus-malus system in force in France entails a reduced penalty if the policyholder is only partially liable for the claim.

Prominent examples of a posteriori ratemaking mechanisms based on several types of events are provided by the experience rating systems in force in North-America. These systems do not only incorporate accidents at fault but also elements of the policyholders' driving record. Let us briefly present the Massachusetts safe driver insurance plan, one of the most sophisticated in force in North America. This program is mandated by state law and encourages safe driving by rewarding drivers who do not cause an accident, or incur a traffic law violation. Specifically, each policyholder is assigned a level between 9 and 35, based on his driving record during the previous 6 years. A new driver begins at level 15 (relativity of 100%). Occupying any level below 15 entails a premium discount, while above level 15 the driver pays a surcharge. For each incident-free year of driving, the policyholder goes down one level. The driver will move up a certain number of levels based on the type of incident: 2 levels for a minor traffic violation, 3 levels for a minor at-fault accident, 4 levels for a major at-fault accident and 5 levels for a major traffic violation. The Massachusetts system forgets all incidents after six years.

This paper addresses the actuarial modelling of such systems, with penalties depending on different types of events. The modelling uses the concept of Markov Chains. We will see that under mild assumptions, the trajectory of each policyholder in the scale can be modelled with the aid of discrete-time Markov processes. The relativities associated to each level will then be computed using the maximum accuracy principle introduced by Norberg (1976). All the reasonings held in this paper are based on the stationary distribution of the system. It is worth to mention that extensions to transient distributions are nevertheless possible in the vein of Borgan et al. (1981).

Let us now detail the contents of the paper. Section 2 presents the actuarial modelling of claim frequencies. In Section 3, we model the policyholders' trajectories in the scale using Markov chains, and we recall some basic features of these stochastic processes. In Section 4, we compute the relativities for each level of the scale. Section 5 presents some numerical illustrations. The final Section 6 concludes.

2 The Model

We consider a portfolio partitioned in κ risk classes $C_1, C_2, \ldots, C_\kappa$ on the basis of the a priori information. Each of these risk classes possesses its own expected annual claim frequency. We denote as λ_k the expected number of claims for policyholders in C_k, $k = 1, 2, \ldots, \kappa$. Furthermore, w_k is the relative weight of the kth risk class in the portfolio.

Let us pick a policyholder at random from the portfolio and denote as N the number of claims he reported during the year. Furthermore, let C be the (unknown) risk class to which this policyholder belongs. Clearly, $\Pr[C = C_k] = w_k$. Denoting as Θ the (unknown) accident proneness of this policyholder, the conditional probability mass function of N is given by

$$\Pr[N = j|\Theta = \theta, C = C_k] = \exp(-\lambda_k\theta)\frac{(-\lambda_k\theta)^j}{j!}, \;\; j \in \mathbb{N} = \{0, 1, 2, \ldots\}.$$

The risk profile of the portfolio is described by the structure function $u(\cdot)$. More formally, $u(\cdot)$ is the probability density function of Θ and we assume that $\mathbb{E}[\Theta] = 1$. Since Θ represents the residual effect of unobserved characteristics, it seems reasonable to assume that Θ and C are mutually independent. Hence, the unconditional probability mass function of N is given by

$$\Pr[N = j] = \sum_k w_k \int_0^{+\infty} \Pr[N = j|\Theta = \theta, C = C_k]u(\theta)d\theta, \;\; j \in \mathbb{N}.$$

We distinguish among τ different types of claims caused by the policyholder. Each type of claims induces a specific penalty for the policyholder. For instance, one could think of

- claims with bodily injuries and claims with material damage only ($\tau = 2$)
- claims with partial liability and claims with full liability ($\tau = 2$)
- introducing claim severities (for instance, claims with amount less than \$ 1 000, between \$ 1 000 and \$ 10 000, and claims above \$ 10 000, so that $\tau = 3$). In this case, we have to assume that claim severities and claim frequencies are mutually independent.

We assume a multinomial scheme for the classification of the claims, and we denote as $q_{k1}, q_{k2}, \ldots, q_{k\tau}$ the probability that the claim is of type $1, 2, \ldots, \tau$, respectively, for a policyholder in risk class C_k. The identity $q_{k1} + q_{k2} + \ldots + q_{k\tau} = 1$ obviously holds true. Now, let $N_1, N_2, \ldots, N_\tau$ be the number of claims of type $1, 2, \ldots, \tau$, respectively. Given Θ and C, the random variables $N_1, N_2, \ldots, N_\tau$ are mutually independent, with respective conditional probability mass function

$$\Pr[N_l = j|\Theta = \theta, C = C_k] = \exp(-\lambda_k\theta q_{kl})\frac{(-\lambda_k\theta q_{kl})^j}{j!}, \;\; j \in \mathbb{N},$$

for $l = 1, \ldots, \tau$.

3 Markov Modelling

Bonus-malus systems can be modeled using Markov chains. This route has been followed for a long time by numerous researchers, such as Norberg (1976), Gilde & Sundt (1989), Denuit & Dhaene (2001), Centeno & Silva (2001) and Pitrebois et al. (2003).

The scale is assumed to have $s + 1$ levels, numbered from 0 to s. A specified level is assigned to a new driver (often according to the use of the vehicle). Each claim free year is rewarded by a bonus point (i.e. the driver goes one level down). Each type of claims entails a specific penalty, expressed as a fixed number of levels per claim.

We assume that the scale possesses the following Markovian property: the knowledge of the present level and of the number of claims of each type filed during the present year suffices to determine the level to which the policy is transferred. This ensures that the bonus-malus system may be represented by a Markov chain (at least conditionally on the observable characteristics and random effects).

Let $p_{\ell_1\ell_2}(\vartheta;\boldsymbol{q})$ be the probability of moving from level ℓ_1 to level ℓ_2 for a policyholder with annual mean claim frequency ϑ and vector probability $\boldsymbol{q}=(q_1,\ldots,q_\tau)^t$; here q_j is the probability that the claim be of type j. Further, $\boldsymbol{M}(\vartheta;\boldsymbol{q})$ is the one-step transition matrix, i.e. $\boldsymbol{M}(\vartheta;\boldsymbol{q})=\{p_{\ell_1\ell_2}(\vartheta;\boldsymbol{q})\}$, $\ell_1,\ell_2=0,1,\ldots,s$. Taking the νth power of $\boldsymbol{M}(\vartheta;\boldsymbol{q})$ yields the ν-step transition matrix whose element $(\ell_1\ell_2)$, denoted as $p^{(\nu)}_{\ell_1\ell_2}(\vartheta;\boldsymbol{q})$, is the probability of moving from level ℓ_1 to level ℓ_2 in ν transitions.

The transition matrix $\boldsymbol{M}(\vartheta;\boldsymbol{q})$ associated to such a bonus-malus system is assumed to be regular, i.e. there exists some integer $\xi_0\geq 1$ such that all entries of $\{\boldsymbol{M}(\vartheta;\boldsymbol{q})\}^{\xi_0}$ are strictly positive. Consequently, the Markov chain describing the trajectory of a policyholder with expected claim frequency ϑ and vector probability $\boldsymbol{q}$ is ergodic and thus possesses a stationary distribution $\boldsymbol{\pi}(\vartheta;\boldsymbol{q})=(\pi_0(\vartheta;\boldsymbol{q}),\pi_1(\vartheta;\boldsymbol{q}),\ldots,\pi_s(\vartheta;\boldsymbol{q}))^t$; $\pi_\ell(\vartheta;\boldsymbol{q})$ is the stationary probability for a policyholder with mean frequency ϑ to be in level ℓ i.e.

$$\pi_{\ell_2}(\vartheta;\boldsymbol{q})=\lim_{\nu\to+\infty}p^{(\nu)}_{\ell_1\ell_2}(\vartheta;\boldsymbol{q}).$$

Let $\boldsymbol{e}$ be a column vector of 1's and let $\boldsymbol{E}$ be the $(s+1)\times(s+1)$ matrix all of whose entries are 1, i.e. consisting of $s+1$ column vectors $\boldsymbol{e}$. Then, the stationary probabilities are directly obtained from the formula

$$\boldsymbol{\pi}^t(\vartheta;\boldsymbol{q})=\boldsymbol{e}^t\big(\mathbf{I}-\boldsymbol{M}(\vartheta;\boldsymbol{q})+\boldsymbol{E}\big)^{-1}$$

that can be found e.g. in Rolski et al. (1999).

Let $L_{\vartheta;\boldsymbol{q}}$ be valued in $\{0,1,\ldots,s\}$ and conform to the distribution $\boldsymbol{\pi}(\vartheta;\boldsymbol{q})$ i.e.

$$\Pr[L_{\vartheta;\boldsymbol{q}}=\ell]=\pi_\ell(\vartheta;\boldsymbol{q}),\;\;\ell=0,1,\ldots,s.$$

The variable $L_{\vartheta;\boldsymbol{q}}$ thus represents the level occupied by a policyholder with annual expected claim frequency ϑ and probability vector $\boldsymbol{q}$ once the steady state has been reached.

Now, let L be the level occupied in the scale by a randomly selected policyholder once the steady state has been reached. The distribution of L can be written as

$$\Pr[L=\ell]=\sum_k w_k\int_0^{+\infty}\pi_\ell(\lambda_k\theta;\boldsymbol{q}_k)u(\theta)d\theta,\;\;\ell=0,1,\ldots,s. \tag{3.1}$$

4 Determination of the Relativities

The relativity associated to level ℓ is denoted as r_ℓ; the meaning is that an insured occupying that level pays an amount of premium equals to r_ℓ% of the reference premium determined on the basis of his observable characteristics.

Following NORBERG (1976), our aim is to minimize the expected squared difference between the "true" relative premium Θ and the relative premium r_L applicable to this policyholder (after the stationary state has been reached), i.e. the goal is to minimize

$$\begin{aligned}\mathbb{E}\Big[(\Theta - r_L)^2\Big] &= \sum_{\ell=0}^{s} \mathbb{E}\Big[(\Theta - r_\ell)^2 \Big| L = \ell\Big] \Pr[L = \ell] \\ &= \sum_k w_k \int_0^{+\infty} \sum_{\ell=0}^{s} (\theta - r_\ell)^2 \pi_\ell(\lambda_k\theta; \boldsymbol{q}_k) u(\theta) d\theta.\end{aligned}$$

The solution is given by

$$r_\ell = \mathbb{E}[\Theta|L = \ell] = \frac{\sum_k w_k \int_0^{+\infty} \theta \pi_\ell(\lambda_k\theta; \boldsymbol{q}_k) u(\theta) d\theta}{\sum_k w_k \int_0^{+\infty} \pi_\ell(\lambda_k\theta; \boldsymbol{q}_k) u(\theta) d\theta}. \quad (4.1)$$

It is easily seen that $\mathbb{E}[r_L] = 1$, resulting in financial equilibrium once steady state is reached.

5 Numerical Illustrations

In this section, we consider an application similar to the one developed in Chapter 13 of LEMAIRE (1995). As mentioned in the introduction, we do not use the Beta modelling of PICARD (1976) and work with bonus-malus scales (and not with credibility formulas). Moreover, we allow explicitly for a priori risk classification.

5.1 A Priori Ratemaking

The data used to illustrate this paper relate to a Belgian motor third party liability portfolio observed during the year 1997. The data set comprises 19,585 policies. The overall mean claim frequency is 19.5% (far above the European average). The annual frequency for claims with bodily injuries is 1.6%; the frequency for claims without bodily injuries (that is, claims with material damage only) amounts to 17.9%.

In this portfolio, the two types of claims we consider are positively correlated. This can be seen from Table 5.1, where the conditional expectation of the number of claim of one type is computed given the number of claims of the other type. The more claim of one type reported, the higher this conditional expectation, resulting in positive dependence.

The following information is available on an individual basis: in addition to the number of claims filed by each policyholder (with the dichotomy bodily injuries/material damage only) and the exposure-to-risk from which these claims originate (i.e. the number of days the policy has been in force during 1997), we know the age of the policyholder in 1997 (18-22 years, 23-30, 31 and above), his/her gender (male-female), the kind of district where he/she lives (rural area or urban area) and the power of the vehicle in kilowatts (less than or above 66 KW).

In this section, the structure function is taken to be a gamma probability density function with unit mean, i.e.

$$u(\theta) = \frac{1}{\Gamma(a)} a^a \theta^{a-1} \exp(-a\theta)\,,\ \theta > 0,$$

Conditional expectation of the number of claims with bodily injuries		Conditional expectation of the number of claims with material damage only	
Given the number of claims with material damage only		Given the number of claims with bodily injuries	
=0	0.014	=0	0.177
=1	0.019	=1	0.255
=2	0.022	=2	0.701

Table 5.1: Expected annual frequency of claims with bodily injuries (resp. with material damage only) given the number of claims with material damage only (resp. with bodily inuries).

for some $a > 0$. A segmented tariff has been built on the basis of a Negative Binomial regression model. More precisely, given $\Theta = \theta$, the annual claim fequency N of the policyholder has probability function

$$\begin{aligned}\Pr[N = k|\Theta = \theta] &= \exp\Big(-d \exp\big(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5\big)\Big) \\ &\quad \times \frac{\Big(d \exp\big(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5\big)\Big)^k}{k!}\end{aligned}$$

for $k = 0, 1, 2, \ldots$, where d is the exposure-to-risk (in years), x_1 equals 1 if the policyholder's age is between 18 and 22, and 0 otherwise, x_2 equals 1 if the policyholder's age is between 23 and 30, and 0 otherwise, x_3 equals 1 if the policyholder is a female, and 0 otherwise, x_4 equals 1 if the policyholder lives in a rural area, and 0 otherwise, and x_5 equals 1 if the vehicle is less than 66 KW, and 0 otherwise. This models has been fitted using the GENMOD procedure of SAS/STAT. All the regression coefficients were found significantly different from 0. This yields the 24 risk classes $C_1, C_2, \ldots, C_{24}$ described in Table 5.2. Table 5.3 gives for each these risk classes C_k the specific annual expected claim frequency λ_k, the relative weight w_k and the probabilities q_{k1} and q_{k2} that the claim cause bodily injuries or not (q_{k1} can be estimated as the ratio of the number of claims with bodily injuries and the total number of claims filed by the policyholders in C_k).

5.2 Bonus-Malus Scale

Let us now consider the soft experience rating system defined in Taylor (1997) with the transition rules formerly used by the Japanese companies. Instead of considering one type of claims we now penalize differently claims with bodily injuries and claims with material damage only. There are 9 levels (numbered from 0 to 8). Level 6 is the starting level. A higher level number indicates a higher premium. If no claims have been reported by the policyholder then he moves one level down. Claims with material damage only are penalized by 2 levels whereas claims with bodily injuries entail a penalty of 4 levels. If n_1 claims with bodily injuries and n_2 claims with material damage only are reported during the year then

Risk Class	Age	Gender	District	Power
C_1	$18 - 22$	Male	Urban	> 66kW
C_2	$18 - 22$	Male	Rural	> 66kW
C_3	$18 - 22$	Male	Urban	< 66kW
C_4	$18 - 22$	Male	Rural	< 66kW
C_5	$23 - 30$	Male	Urban	> 66kW
C_6	$23 - 30$	Male	Rural	> 66kW
C_7	$23 - 30$	Male	Urban	< 66kW
C_8	$23 - 30$	Male	Rural	< 66kW
C_9	> 30	Male	Urban	> 66kW
C_{10}	> 30	Male	Rural	> 66kW
C_{11}	> 30	Male	Urban	< 66kW
C_{12}	> 30	Male	Rural	< 66kW
C_{13}	$18 - 22$	Female	Urban	> 66kW
C_{14}	$18 - 22$	Female	Rural	> 66kW
C_{15}	$18 - 22$	Female	Urban	< 66kW
C_{16}	$18 - 22$	Female	Rural	< 66kW
C_{17}	$23 - 30$	Female	Urban	> 66kW
C_{18}	$23 - 30$	Female	Rural	> 66kW
C_{19}	$23 - 30$	Female	Urban	< 66kW
C_{20}	$23 - 30$	Female	Rural	< 66kW
C_{21}	> 30	Female	Urban	> 66kW
C_{22}	> 30	Female	Rural	> 66kW
C_{23}	> 30	Female	Urban	< 66kW
C_{24}	> 30	Female	Rural	< 66kW

Table 5.2: Description of the 24 risk classes composing the portfolio.

Risk Class	Weight w_k	Frequency λ_k	Probability q_{k1}	Probability q_{k2}
C_1	0.0009	0.4138	0.0891	0.9109
C_2	0.0030	0.3323	0.0835	0.9165
C_3	0.0036	0.3909	0.0949	0.9051
C_4	0.0115	0.3140	0.0890	0.9110
C_5	0.0217	0.2811	0.0823	0.9177
C_6	0.0526	0.2257	0.0771	0.9229
C_7	0.0477	0.2655	0.0877	0.9123
C_8	0.0874	0.2132	0.0822	0.9178
C_9	0.0613	0.2082	0.0746	0.9254
C_{10}	0.1308	0.1672	0.0699	0.9301
C_{11}	0.0666	0.1967	0.0796	0.9204
C_{12}	0.1510	0.1580	0.0746	0.9254
C_{13}	0.0004	0.3874	0.0946	0.9054
C_{14}	0.0006	0.3111	0.0887	0.9113
C_{15}	0.0027	0.3660	0.1007	0.8993
C_{16}	0.0075	0.2940	0.0945	0.9055
C_{17}	0.0067	0.2632	0.0874	0.9126
C_{18}	0.0133	0.2113	0.0819	0.9181
C_{19}	0.0336	0.2486	0.0931	0.9069
C_{20}	0.0797	0.1997	0.0873	0.9127
C_{21}	0.0230	0.1949	0.0793	0.9207
C_{22}	0.0404	0.1565	0.0743	0.9257
C_{23}	0.0505	0.1842	0.0846	0.9154
C_{24}	0.1035	0.1479	0.0792	0.9208

Table 5.3: Expected annual claim frequencies for the 24 risk classes described in Table 5.2, and proportion of claims with and without bodily injuries.

the policyholder moves $4n_1 + 2n_2$ levels up. This system is abbreviated as -1/+2/+4, in obvious notations.

The transition matrix for a policyholder with annual mean claim frequency ϑ and vector probability $\boldsymbol{q} = (q_1, q_2)^t$ is given by

$$\boldsymbol{M}(\vartheta; \boldsymbol{q}) = \begin{pmatrix} P_0 & 0 & P_1 & 0 & P_2 & 0 & P_3 & 0 & 1-\Sigma \\ P_0 & 0 & 0 & P_1 & 0 & P_2 & 0 & P_3 & 1-\Sigma \\ 0 & P_0 & 0 & 0 & P_1 & 0 & P_2 & 0 & 1-\Sigma \\ 0 & 0 & P_0 & 0 & 0 & P_1 & 0 & P_2 & 1-\Sigma \\ 0 & 0 & 0 & P_0 & 0 & 0 & P_1 & 0 & 1-\Sigma \\ 0 & 0 & 0 & 0 & P_0 & 0 & 0 & P_1 & 1-\Sigma \\ 0 & 0 & 0 & 0 & 0 & P_0 & 0 & 0 & 1-\Sigma \\ 0 & 0 & 0 & 0 & 0 & 0 & P_0 & 0 & 1-\Sigma \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_0 & 1-\Sigma \end{pmatrix}$$

where

$$\begin{aligned} P_0 &= \exp(-\vartheta) \\ P_1 &= \vartheta q_2 \exp(-\vartheta q_2) \exp(-\vartheta q_1) \\ P_2 &= \frac{(\vartheta q_2)^2}{2} \exp(-\vartheta q_2) \exp(-\vartheta q_1) + \exp(-\vartheta q_2) \vartheta q_1 \exp(-\vartheta q_1) \\ P_3 &= \frac{(\vartheta q_2)^3}{3!} \exp(-\vartheta q_2) \exp(-\vartheta q_1) + \vartheta q_2 \exp(-\vartheta q_2) \vartheta q_1 \exp(-\vartheta q_1) \end{aligned}$$

and Σ represents the sum of all the elements in the same row.

5.3 Computation of the Relativities

Table 5.4 gives for each of the 9 levels of the bonus-malus scale the proportion of the portfolio in that level (column 2) and the relativity attached to that level (column 3) for the system -1/+2/+4. About half of the portfolio is in level 0 and enjoys a discount of about 30%. The rest of the portfolio is spread out among levels 1-8. The r_ℓ's range from 69.38% to 209.82%.

In order to compare the results with those of traditional bonus-malus scales, we have also considered three other scales. For all of them, each claim-free year is rewarded by one level down in the scale. The first bonus-malus system penalizes each claim (with or without bodily injuries) by 2 levels up in the scale (this system is referred to as -1/+2), the second one by 3 levels up (this system is referred to as -1/+3) and the third one by 4 levels up (this system is referred to as -1/+4). The r_ℓ's for these scales are computed with the formulas derived in Pitrebois et al. (2003).

Clearly, the more the claims are penalized, the more the policyholders occupying the lowest levels are awarded discounts, and the less the policyholders in the upper part of the scale are penalized. The scale -1/+2/+4 is closer to scale -1/+2. This is because the majority of the claims only induce material damage. Nevertheless, r_8 is reduced from 218.03% to 209.82% when the claims with bodily injuries are more severely penalized.

	-1/+2/+4		-1/+2		-1/+3		-1/+4	
Level ℓ	π_ℓ	r_ℓ	π_ℓ	r_ℓ	π_ℓ	r_ℓ	π_ℓ	r_ℓ
8	4.67%	209.82%	4.09%	218.03%	7.44%	187.55%	10.37%	169.50%
7	4.04%	190.04%	3.55%	197.68%	6.16%	170.10%	8.49%	152.90%
6	4.05%	169.06%	3.55%	176.73%	6.14%	148.81%	7.16%	139.75%
5	3.96%	155.00%	3.55%	161.55%	5.68%	136.07%	6.16%	129.11%
4	5.21%	133.41%	4.72%	139.85%	5.24%	126.22%	8.77%	104.65%
3	5.12%	124.99%	4.85%	129.73%	8.88%	101.98%	7.21%	98.96%
2	9.57%	103.52%	10.07%	104.74%	7.34%	96.95%	6.00%	93.89%
1	7.89%	98.66%	8.29%	99.83%	6.13%	92.40%	5.05%	89.35%
0	55.50%	69.38%	57.33%	70.36%	46.99%	64.38%	40.79%	61.34%

Table 5.4: Results for the bonus-malus systems -1/+2/+4, -1/+2, -1/+3 and -1/+4.

5.4 Impact of the Average Claim Frequency

The numerical example discussed in Sections 5.1-5.3 uses the portfolio of a company that exhibits an average claim frequency much above the standard ones (that typically range in the interval 6%-10%). This of course produces penalties that are not too high, as it can be seen from Table 5.4.

The effect of the average claim frequency at the portfolio level on the relativities is explored in this section. More precisely, we have kept the 24 risk classes described in Table 5.2 but we have scaled the corresponding annual claim frequency to produce an average claim frequency of 6, 8 and 10%, respectively, at the portfolio level.

The results are displayed in Table 5.5. We can see there that when the average claim frequency increases from 6% to 10%, the proportion of policyholders in level 0 decreases from 85.93% to 76.01%. Moreover, the maluses increase as the average claim frequency of the portfolio decreases (the relativity r_8 decreases from 264.38% to 247.16% as the average claim frequency increases from 6 to 10%). The penalties induced by the BMS remain nevertheless applicable in practice.

6 Conclusion

European directives have introduced complete rating freedom: Insurance companies operating in most EU countries are now free to set up their own rates, select their own classification variables and design their own BMS. In most European countries, companies have taken advantage of this freedom by introducing more rating variables. It can be expected that they will start to compete on the basis of BMS. In that respect, this paper offers an alternative approach to traditional bonus-malus scales.

	Freq=0.06		Freq=0.08		Freq=0.10	
Level ℓ	π_ℓ	r_ℓ	π_ℓ	r_ℓ	π_ℓ	r_ℓ
8	0.14%	264.38%	0.37%	256.53%	0.76%	247.16%
7	0.19%	247.95%	0.45%	239.01%	0.85%	229.13%
6	0.35%	217.70%	0.71%	211.61%	1.18%	203.86%
5	0.48%	206.90%	0.89%	199.30%	1.40%	190.78%
4	1.28%	170.84%	1.94%	166.92%	2.63%	161.39%
3	1.48%	167.23%	2.19%	161.48%	2.88%	154.78%
2	5.29%	138.35%	6.58%	133.14%	7.59%	127.60%
1	4.87%	134.93%	5.91%	129.13%	6.69%	123.22%
0	85.93%	91.78%	80.95%	88.34%	76.01%	84.80%

Table 5.5: Results for the BMS -1+/+2/+4 with annual expected claim frequency of 6, 8 and 10%, respectively, at the portfolio level.

Aknowledgements

The authors thank the referees for interesting comments which led to considerable improvements of this paper. The authors gratefully acknowledge the financial support of Belgian Government under the "Projet d'Action de Recherches Concertées" 04/09-320.

References

[1] Borgan, Ø, Hoem, J.M., and Norberg, R. (1981). A nonasymptotic criterion for the evaluation of automobile bonus systems. *Scandinavian Actuarial Journal*, 265-178.

[2] Centeno, M., and Silva, J.M.A. (2001). Bonus systems in an open portfolio. *Insurance: Mathematics & Economics* **28**, 341-350.

[3] Denuit, M., and Dhaene, J. (2001). Bonus-malus scales using exponential loss functions. *German Actuarial Bulletin* **25**, 13-27.

[4] Dionne, G., and Vanasse, C. (1989). A generalization of actuarial automobile insurance rating models: the Negative Binomial distribution with a regression component. *ASTIN Bulletin* **19**, 199-212.

[5] Dionne, G., and Vanasse, C. (1992). Automobile insurance ratemaking in the presence of asymmetrical information. *Journal of Applied Econometrics* **7**, 149-165.

[6] Gilde, V., and Sundt, B. (1989). On Bonus systems with credibility scales. *Scandinavian Actuarial Journal*, 13-22

[7] Kaas, R., Goovaerts, M.J., Dhaene, J., & Denuit, M. (2001). *Modern Actuarial Risk Theory*. Kluwer Academic Publishers, Dordrecht.

[8] Lemaire, J. (1995). *Bonus-Malus Systems in Automobile Insurance*. Kluwer Academic Publisher, Boston.

[9] Norberg, R. (1976). A credibility theory for automobile bonus system. *Scandinavian Actuarial Journal*, 92-107.

[10] Picard, P. (1976). Généralisation de l'étude sur la survenance des sinistres en assurance automobile. *Bulletin Trimestriel de l'Institut des Actuaires Français*, 204-267.

[11] Pinquet, J. (2000). Experience rating through heterogeneous models. In *Handbook of Insurance*, edited by G. Dionne. Kluwer Academic Publishers.

[12] Pitrebois, S., Denuit, M., & Walhin, J.-F. (2003). Setting a BMS in the presence of other rating factors: Taylor's work revisited. *ASTIN Bulletin* **33**, 419-436.

[13] Rolski, T., Schmidli, H., Schmidt, V., and Teugels, J. (1999). *Stochastic Processes for Insurance and Finance*. John Wiley & Sons, New York.

[14] Taylor, G. (1997). Setting a Bonus-Malus scale in the presence of other rating factors. *ASTIN Bulletin* **27**, 319-327.