

On the Pricing of Top & Drop Excess of Loss Covers

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Agenda

- ⌘ **Definition**
- ⌘ **Numerical examples**
- ⌘ **Modelization**
- ⌘ **Exact solution**
- ⌘ **False solution**
- ⌘ **Approximate solution**
- ⌘ **Conclusion**

Top & Drop covers : definition

⌘ Top layer covered

⌘ Cover may drop to a lower layer

Numerical example 1

⌘ Layer 1 : 300 XS 100

⌘ Layer 2 : 400 XS 400

⌘ Assume the cedent fears

1) Big losses > 800

2) Accumulation of losses > 20 and < 100

Numerical example 1

⌘ 200 XS 800

⌘ AND / OR

⌘ 200 XS 200 aggregate for claims > 20
fgu with max 100 per claim

⌘ No reinstatement

Numerical example 2

⌘ Layer 1 : 200 XS 200 with one reinstatement

⌘ Layer 2 : 400 XS 400

⌘ Assume the cedent fears

1) Big losses > 800

2) Accumulation of claims in layer 1

Numerical example 2

⌘ 200 XS 800

⌘ AND / OR

⌘ 200 XS 200

⌘ with an annual aggregate deductible =
400

⌘ Unlimited free reinstatements

Modelization

⌘ **X : large claims**

⌘ **N : number of large claims**

⌘ **Y : small claims**

⌘ **M : number of small claims**

⌘ **Classical hypothesis of independence**

Panjer's algorithm

$$S = f(X_1) + \dots + f(X_N)$$

$$f_i = P(f(X) = i)$$

$$g_s = P(S = s)$$

$$g_0 = e^{-\lambda(1-f_0)}$$

$$g_s = \frac{\lambda}{s} \sum_{i=1}^s i f_i g_{s-i}$$

Panjer's algorithm

Easy to obtain the distribution of

$$S = h_1(X_1) + \dots + h_1(X_N)$$

and

$$T = h_2(X_1) + \dots + h_2(X_N)$$

and

$$U = h_3(Y_1) + \dots + h_3(Y_N)$$

Example 1

$$X_i^{\text{Re-top}} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\text{Re-drop}} = \min(100, X_i I_{X_i \geq 20})$$

$$Y_i^{\text{Re-drop}} = \min(100, Y_i I_{Y_i \geq 20})$$

Example 1

$$S = X_1^{\text{Re-top}} + \dots + X_N^{\text{Re-top}}$$

$$T = X_1^{\text{Re-drop}} + \dots + X_N^{\text{Re-drop}}$$

$$U = Y_1^{\text{Re-drop}} + \dots + Y_M^{\text{Re-drop}}$$

$$\text{Cover} = \min(200, S + \max(0, T + U - 200))$$

Example 2

$$X_i^{\text{Re-top}} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\text{Re-drop}} = \min(200, \max(0, X_i - 200))$$

$$Y_i^{\text{Re-drop}} = \min(200, \max(0, Y_i - 200))$$

Example 2

$$S = X_1^{\text{Re-top}} + \dots + X_N^{\text{Re-top}}$$

$$T = X_1^{\text{Re-drop}} + \dots + X_N^{\text{Re-drop}}$$

$$U = Y_1^{\text{Re-drop}} + \dots + Y_M^{\text{Re-drop}}$$

$$\text{Cover} = \max(0, S + T + U - 400)$$

Bivariate Panjer's algorithm

- ⌘ Let N belong to Panjer's family of distributions
- ⌘ Let X and Y be possibly dependent
- ⌘ Let $S = X_1 + \dots + X_N$
- ⌘ Let $T = Y_1 + \dots + Y_N$
- ⌘ Then a multivariate version of Panjer's algorithm exists

Bivariate Panjer's algorithm (Poisson case)

$$g_{0,0} = e^{-\lambda(1-f_{0,0})}$$

$$g_{s,t} = \lambda \sum_{x,y}^{s,t} \frac{x}{s} g_{s-x,t-y} f_{x,y}, \quad s \geq 1$$

$$g_{s,t} = \lambda \sum_{x,y}^{s,t} \frac{y}{t} g_{s-x,t-y} f_{x,y}, \quad t \geq 1$$

Numerical example

⌘ Severity : limited Pareto

⌘ Frequency : Poisson

⌘ Large claims : $\lambda = 0.3$, $A=400$,
 $B=1000$, $\alpha=0.9$

⌘ Small claims : $\lambda=2.5$, $A=20$,
 $B=400$, $\alpha = 1.4$

Limited Pareto distribution

$$\begin{aligned} F(x) &= 0, \quad x < A \\ &= \frac{A^{-\alpha} - x^{-\alpha}}{A^{-\alpha} - B^{-\alpha}}, \quad A < x < B \\ &= 1, \quad x > B \end{aligned}$$

Local moment matching with one moment

$$X \prec_{cx} X^{dis}$$

Discretisation of the limited Pareto distribution

$$\begin{aligned}
 f_A &= 1 - \frac{\frac{(A+h)^{1-\alpha}}{1-\alpha} - \frac{A^{1-\alpha}}{1-\alpha} - B^{-\alpha}h}{h(A^{-\alpha} - B^{-\alpha})} \\
 f_{A+jh} &= \frac{2(A+jh)^{1-\alpha} - (A+(j-1)h)^{1-\alpha} - (A+(j+1)h)^{1-\alpha}}{h(1-\alpha)(A^{-\alpha} - B^{-\alpha})} \\
 f_B &= 1 - f_A - f_{A+h} - \dots - f_{B-h}
 \end{aligned}$$

Exact pure premiums

⌘ **Example 1 : $E(\text{Cover}) = 20.519$**

⌘ **Example 2 : $E(\text{Cover}) = 2.252$**

Assumption of independence

⌘ **Example 1 : $21.131 > 20.519$
(conservative)**

⌘ **Example 2 : $1.153 < 2.252$ (not
conservative)**

Fréchet space and theorem

⌘ $R(F_1, F_2)$: space of all random vectors
(distribution F_{12}) with fixed marginals
 F_1 and F_2

⌘ Let $F^{\min} = \max[F_1 + F_2 - 1, 0]$

⌘ Let $F^{\max} = \min[F_1, F_2]$

⌘ Then $F^{\min} \leq F_{12} \leq F^{\max}$

Correlation order

$$\text{⌘}(\mathbf{X}_1, \mathbf{X}_2) <_c (\mathbf{Y}_1, \mathbf{Y}_2) \text{ iif } \mathbf{F}_{\mathbf{X}_1, \mathbf{X}_2} \leq \mathbf{F}_{\mathbf{Y}_1, \mathbf{Y}_2}$$

Fréchet bounds

⌘ Using some lemmas and Fréchet theorem we arrive at

⌘ Example 1 : $19.469 < E[\text{cover}] < 21.279$

⌘ Example 2 : $0.952 < E[\text{cover}] < 5.471$

Assumption of independence

⌘ Using some lemmas we arrive at

Example 1 : 21.131 (better upper bound)

Example 2 : 1.153 (better lower bound)

Summary

⌘ **Example 1 : 19.469 < 20.519 (exact) < 21.131 (indep) < 21.279**

⌘ **Example 2 : 0.952 < 1.153 (indep) < 2.252 (exact) < 5.471**

Extension to dimension higher than 2

- ⌘ Correlation order extends to supermodular order.**
- ⌘ Example 2 may be treated within that framework.**
- ⌘ However, example 1 may not : a specific framework was necessary in dimension 2.**

Conclusion

- ⌘ **Danger of falsely assumed independence.**
- ⌘ **Existing bounds may be crude.**
- ⌘ **Exact model is time-consuming but provides an exact solution.**

Second conclusion

- ⌘ Dependencies is an “à la mode” subject
- ⌘ Copulas
- ⌘ Fréchet bounds
- ⌘ Sometimes exact models exist
- ⌘ Essentially where dependence is induced by contractual cover
- ⌘ Especially in reinsurance world

Congress

⌘ Please visit

www.stat.ucl.ac.be/Samos2002/mainpage.html

Scientific publications

⌘ **Belgian Actuarial Bulletin**

Please visit

www.stat.ucl.ac.be/BAB/