# On the Pricing of Top & Drop Excess of Loss Covers

**Jean-François Walhin**

**Secura Belgian Re**

**Université Catholique de Louvain**

# Agenda

- ⌘ Definition
- ⌘ Numerical examples
- ⌘ Modelization
- ⌘ Exact solution
- ⌘ False solution
- ⌘ Approximate solution
- ⌘ Conclusion

# Top & Drop covers : definition

- ⌘Top layer covered
- ⌘Cover may drop to a lower layer

# Numerical example 1

- **Layer 1 : 300 XS 100**
- **Layer 2 :  400 XS 400**
- **Assume the cedent fears**
  **1) Big losses > 800**
  **2) Accumulation of losses >20 and < 100**

# Numerical example 1

- **200 XS 800**
- **OR**
- **200 XS 200 aggregate for claims > 20 fgu with max 100 per claim**
- **No reinstatement**

# Numerical example 2

- ⌘Layer 1 : 200 XS 200 with one reinstatement
- ⌘Layer 2 : 400 XS 400
- ⌘Assume the cedent fears
  1) Big losses > 800
  2) Accumulation of claims in layer 1

# Numerical example 2

- **200 XS 800**
- **OR**
- **200 XS 200**
- **with an annual aggregate deductible = 400**
- **Unlimited free reinstatements**

# Modelization

- ⌘X : large claims
- ⌘N : number of large claims
- ⌘Y : small claims
- ⌘M : number of small claims
- ⌘Mutual independence

# Example 1

$$X_i^{\mathrm{Re}-top} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\mathrm{Re}-drop} = \min(100, X_i I_{X_i \geq 20})$$

$$Y_i^{\mathrm{Re}-drop} = \min(100, Y_i I_{Y_i \geq 20})$$

# Example 1

$$
\begin{aligned}
S &= X_1^{\mathrm{Re}-top} + \ldots + X_N^{\mathrm{Re}-top} \\
T &= X_1^{\mathrm{Re}-drop} + \ldots + X_N^{\mathrm{Re}-drop} \\
U &= Y_1^{\mathrm{Re}-drop} + \ldots + Y_M^{\mathrm{Re}-drop} \\
Cover &= \min(200, S + \max(0, T + U - 200))
\end{aligned}
$$

# Example 2

$$X_i^{\mathrm{Re}-top} = \min(200, \max(0, X_i - 800))$$

$$X_i^{\mathrm{Re}-drop} = \min(200, \max(0, X_i - 200))$$

$$Y_i^{\mathrm{Re}-drop} = \min(200, \max(0, Y_i - 200))$$

# Example 2

$$
\begin{aligned}
S &= X_1^{\mathrm{Re}-top} + \ldots + X_N^{\mathrm{Re}-top} \\
T &= X_1^{\mathrm{Re}-drop} + \ldots + X_N^{\mathrm{Re}-drop} \\
U &= Y_1^{\mathrm{Re}-drop} + \ldots + Y_M^{\mathrm{Re}-drop} \\
Cover &= \max(0, S + T + U - 400)
\end{aligned}
$$

# Multivariate Panjer's algorithm

- Let N belong to Panjer's family of distributions
- Let X and Y be possibly dependent
- Let $S=X_1+\ldots+X_N$
- Let $T=Y_1+\ldots+Y_N$
- Then a multivariate version of Panjer's algorithm exists

# Numerical example

- **Severity : limited Pareto**
- **Frequency : Poisson**
- **Large claims : lambda = 0.3, A=400, B=1000, alpha=0.9**
- **Small claims : lambda=2.5, A=20, B=400, alpha = 1.4**

# Exact pure premiums

- **Example 1 : 20.519**
- **Example 2 : 2.252**

# Assumption of independence

- **Example 1 : 21.131 > 20.519 (conservative)**
- **Example 2 : 1.153 < 2.252 (not conservative)**

# Fréchet space and theorem

- $R(F_1,F_2)$ : space of all random vectors (distribution $F_{12}$) with fixed marginals $F_1$ and $F_2$
- Let $F^{min}=max[F_1+F_2-1,0]$
- Let $F^{max}=min[F_1,F_2]$
- Then $F^{min}<=F_{12}<=F^{max}$

# Correlation order

⌘ $(X_1,X_2) <_c (Y_1,Y_2)$ iif $F_{X1,X2} <= F_{Y1,Y2}$

# Fréchet bounds

- ⌘Using some lemmas and Fréchet theorem we arrive at
- ⌘Example 1 : 19.469 < E[cover] < 21.279
- ⌘Example 2 : 0.952 < E[cover] < 5.471

# Assumption of independence

- **Using some lemmas we arrive at**
  **Example 1 : 21.131 (better upper bound)**
  **Example 2 : 1.153 (better lower bound)**

# Summary

- **Example 1 : 19.469 < 20.519 (exact) < 21.131 (indep) < 21.279**
- **Example 2 : 0.952 < 1.153 (indep) < 2.252 (exact) < 5.471**

# Extension to dimension higher than 2

- Correlation order extends to supermodular order.
- Example 2 may be treated within that framework.
- However, example 1 may not : a specific framework was necessary in dimension 2.

# Conclusion

- ⌘Danger of falsely assumed independence.
- ⌘Existing bounds may be crude.
- ⌘Exact model is time-consuming but provides an exact solution.