The practical pricing of XL treaties : Actuarial, financial, economic and commercial aspects

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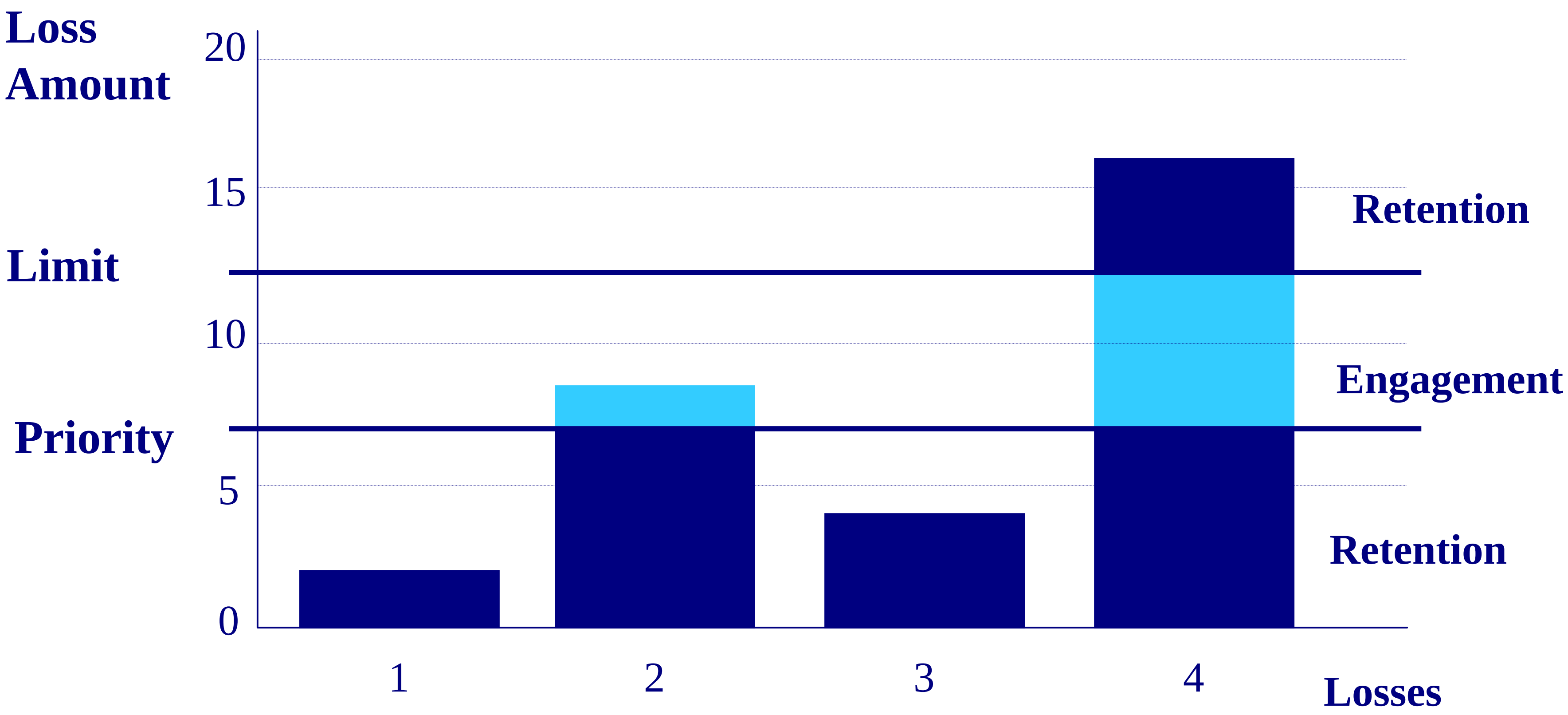
Excess of loss



Excess of loss

- We will price the cover : L xs D.
- Let X_i be a loss.
- Then $R_i=\min(L,\max(0,X_i-D))$ is the reinsurer's loss.
- $S=R_1+\ldots R_N$ is the aggregate claims of the reinsurer (on a yearly basis say).

Panjer's algorithm

- ⌘**N will be assumed to be Poisson distributed.**
- ⌘**Panjers' algorithm is then applicable :**

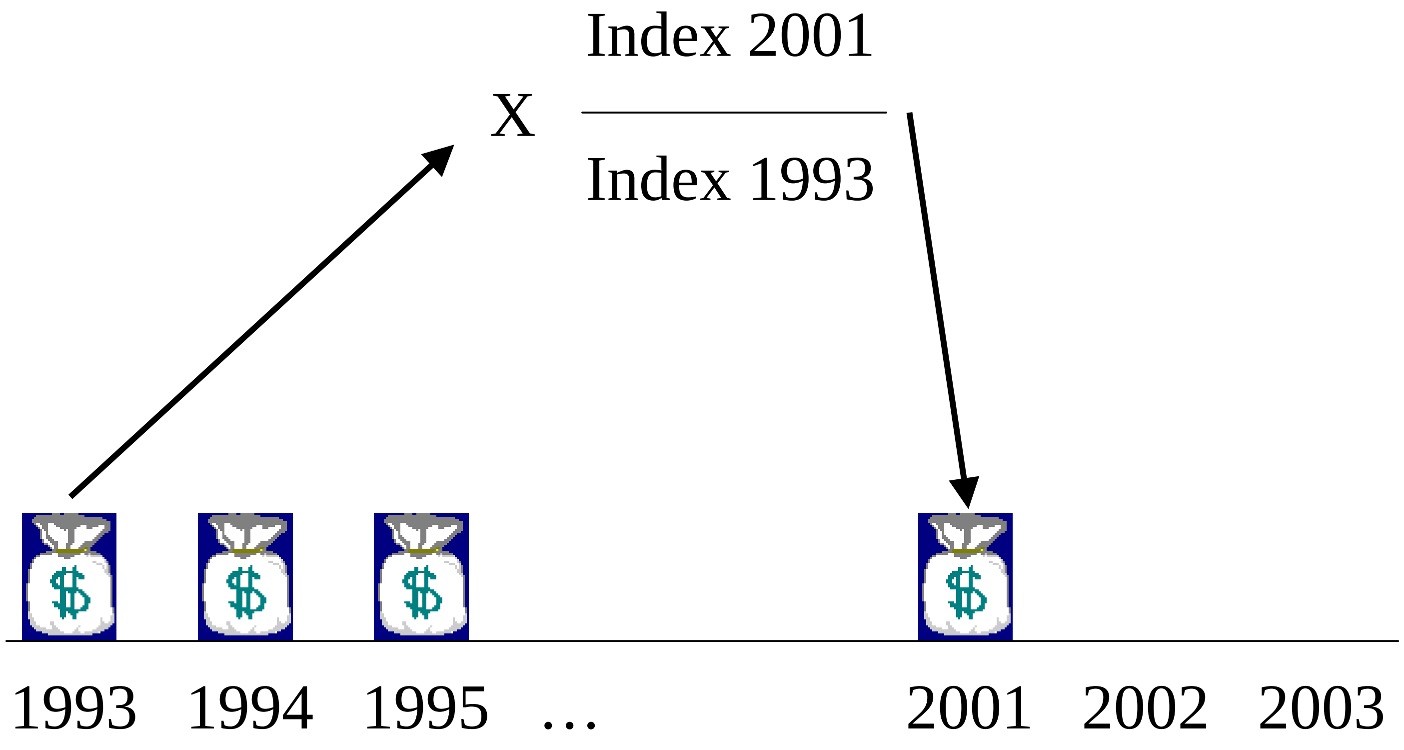
$$
\begin{aligned}
f_S(0) &= e^{-\lambda(1-f_R(0))} \\
f_S(s) &= \lambda \sum_{i=1}^{s} \frac{i}{s} f_R(i) f_S(s-i), s=1,2,...
\end{aligned}
$$

Practical issues

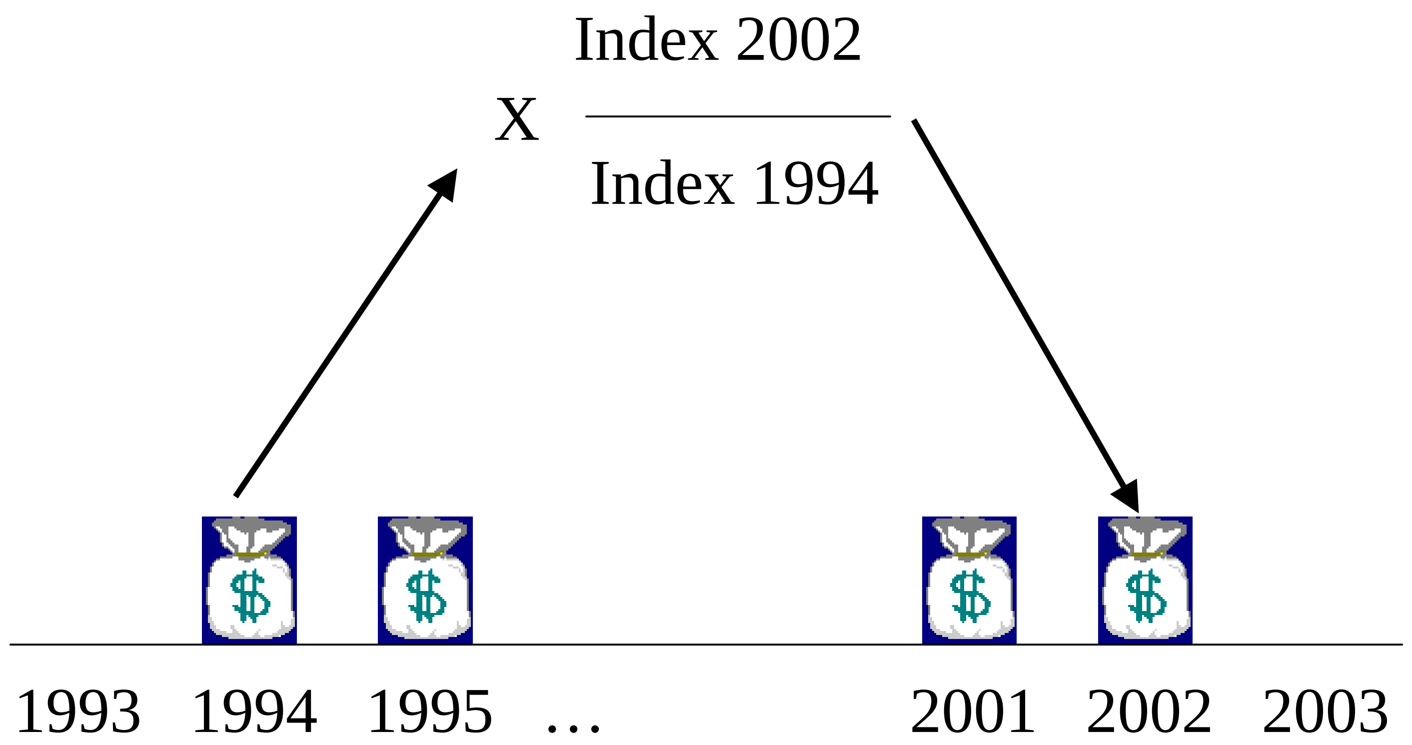
Input : data from the past :
t-n, ..., t-2, (t-1)

Output : pricing for the year t

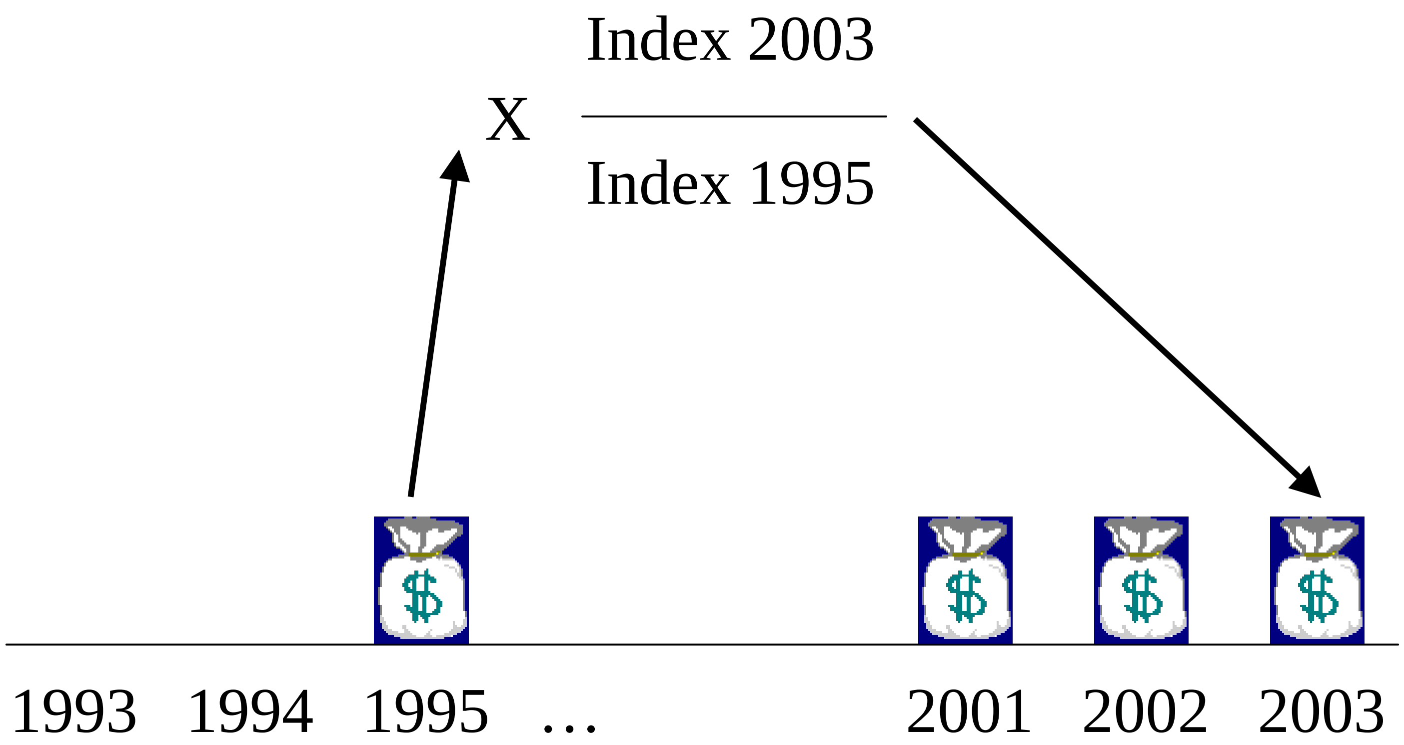
Loss indexation (old)



Loss indexation (old)



Loss Indexation (old)



Original loss amounts ...

Accident	Development							
	1	2	3	4	5	6	7	8
1991	1991	1992	1993	1994	1995	1996	1997	1998
1992	1992	1993	1994	1995	1996	1997	1998	
1993	1993	1994	1995	1996	1997	1998		
1994	1994	1995	1996	1997	1998			
1995	1995	1996	1997	1998				
1996	1996	1997	1998					
1997	1997	1998						
1998	1998							

… become indexed loss amount

Accident	Development							
	1	2	3	4	5	6	7	8
2000	2000	2001	2002	2003	2004	2005	2006	2007
2000	2000	2001	2002	2003	2004	2005	2006	
2000	2000	2001	2002	2003	2004	2005		
2000	2000	2001	2002	2003	2004			
2000	2000	2001	2002	2003				
2000	2000	2001	2002					
2000	2000	2001						
2000	2000							

Extrapolation

	1	2	3	4	…	8
1991	X	X	X	X		X
…						
1996	X	X	X	X		?
1997	X	X	X	?		?
1998	X	X	?	?		?
1998	X	?	?	?		?

Difficulties

- ⌘Estimating inflation (e.g. wage index).
- ⌘Estimating super-imposed inflation.
- ⌘Inflating technical reserves.
- ⌘Changes in the way of reserving.

Particularities of current pricing

- Fitting of distributions of losses including inflation : the parameter of X depends on inflation !
- Analyse of the aggregate claims within the layer.
- Clauses, if any, are applied on the original data : the parameter of X depends on the clauses !

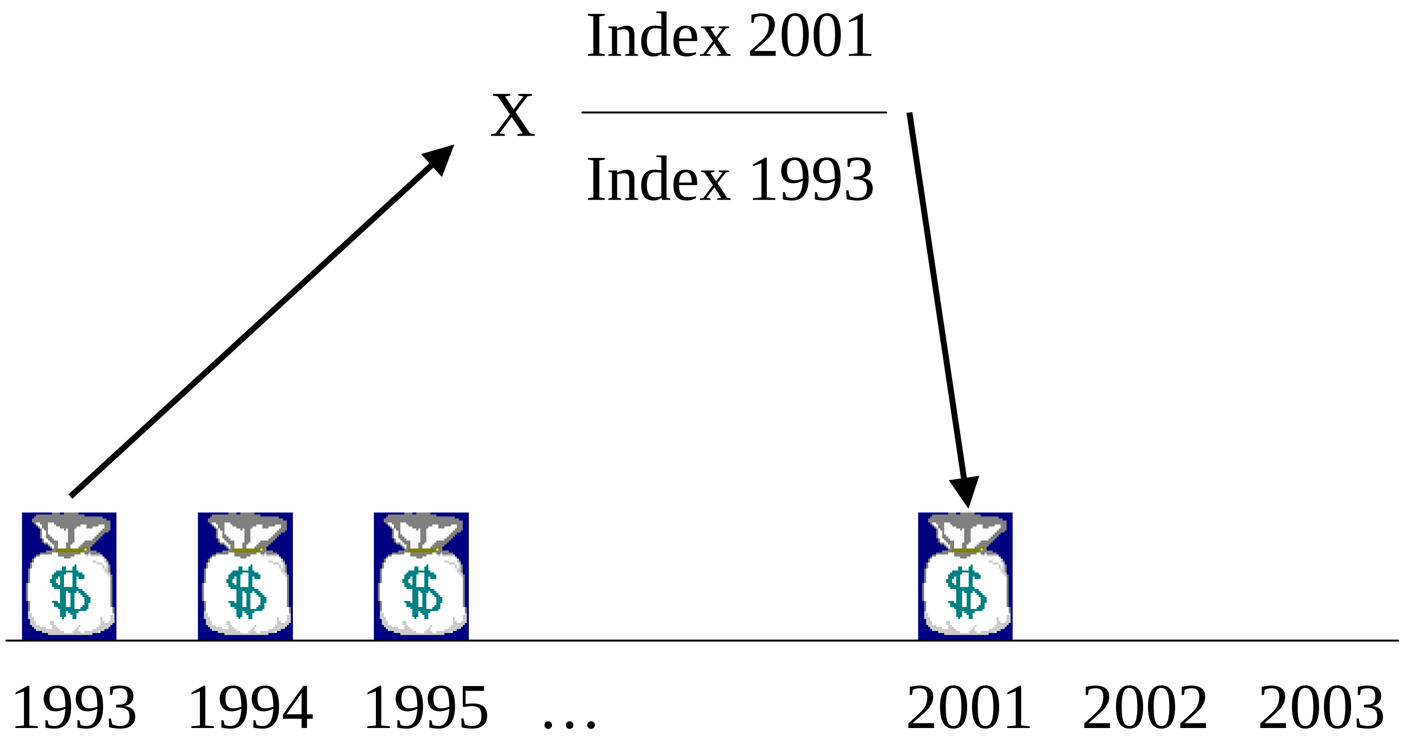
Changing these particularities

- **Each year the fitting has to be redone due to the changes in inflation.**
- **Better solution : do not take into account of the future inflation. Fit the distribution of X once. After that is suffices to inflate the realizations of X for future inflation.**

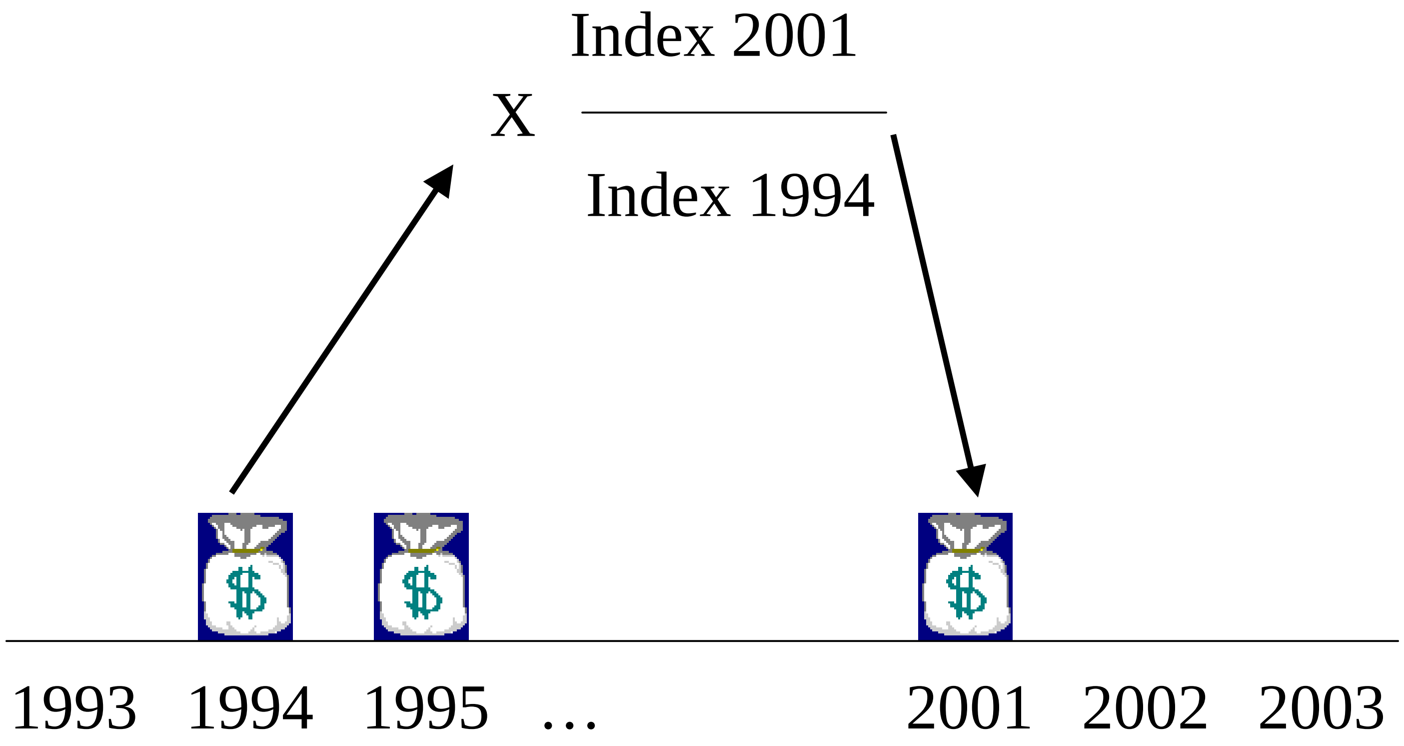
Changing these particularities

⌘ **Moreover the distribution remains the same for subsequent underwriting years. It suffices to correctly inflate for the future. Thus if data has not changed dramatically, a new fit is not necessary.**

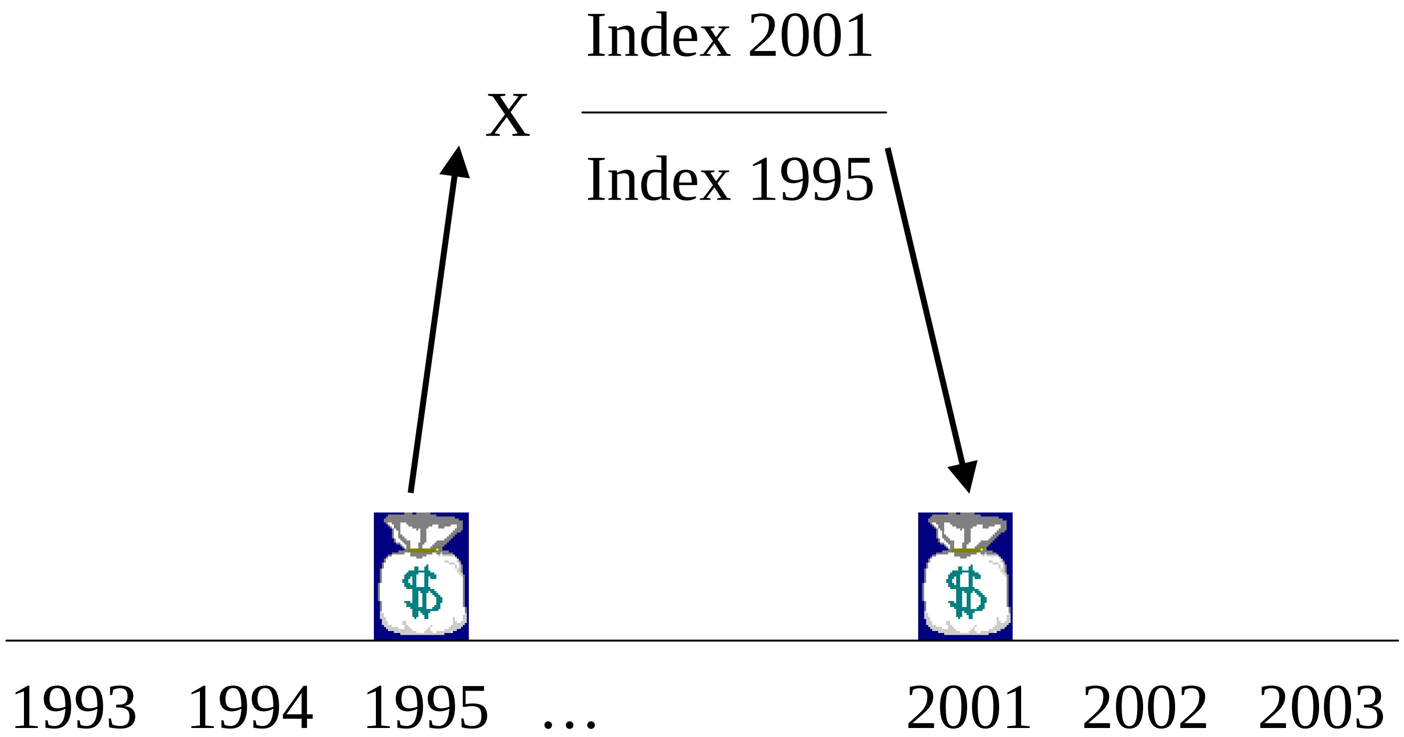
Loss indexation (new)



Loss indexation (new)



Loss Indexation (new)



Indexed loss amount (new)

Accident	Development							
	1	2	3	4	5	6	7	8
2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	
2000	2000	2000	2000	2000	2000	2000		
2000	2000	2000	2000	2000	2000			
2000	2000	2000	2000	2000				
2000	2000	2000	2000					
2000	2000	2000						
2000	2000							

Changing the particularities

- **Analysing different layers on an aggregate basis is surprising.**
- **Different fitting may be obtained for the distribution of min(L,max(0,X-D)) which may not be coherent with the fact that they all depend on the same underlying X.**
- **We therefore prefer to work on the losses X_i from ground up.**

Changing the particularities

- If the distribution of this random variable is estimated then each layer may be priced homogeneously.
- Moreover triangles (the problem of extrapolation) are more stable.

Changing the particularities

- Clauses are not applied on the original data.
- We therefore fit the distribution of the real loss.
- If there is a clause it will be applied on the loss afterwards.

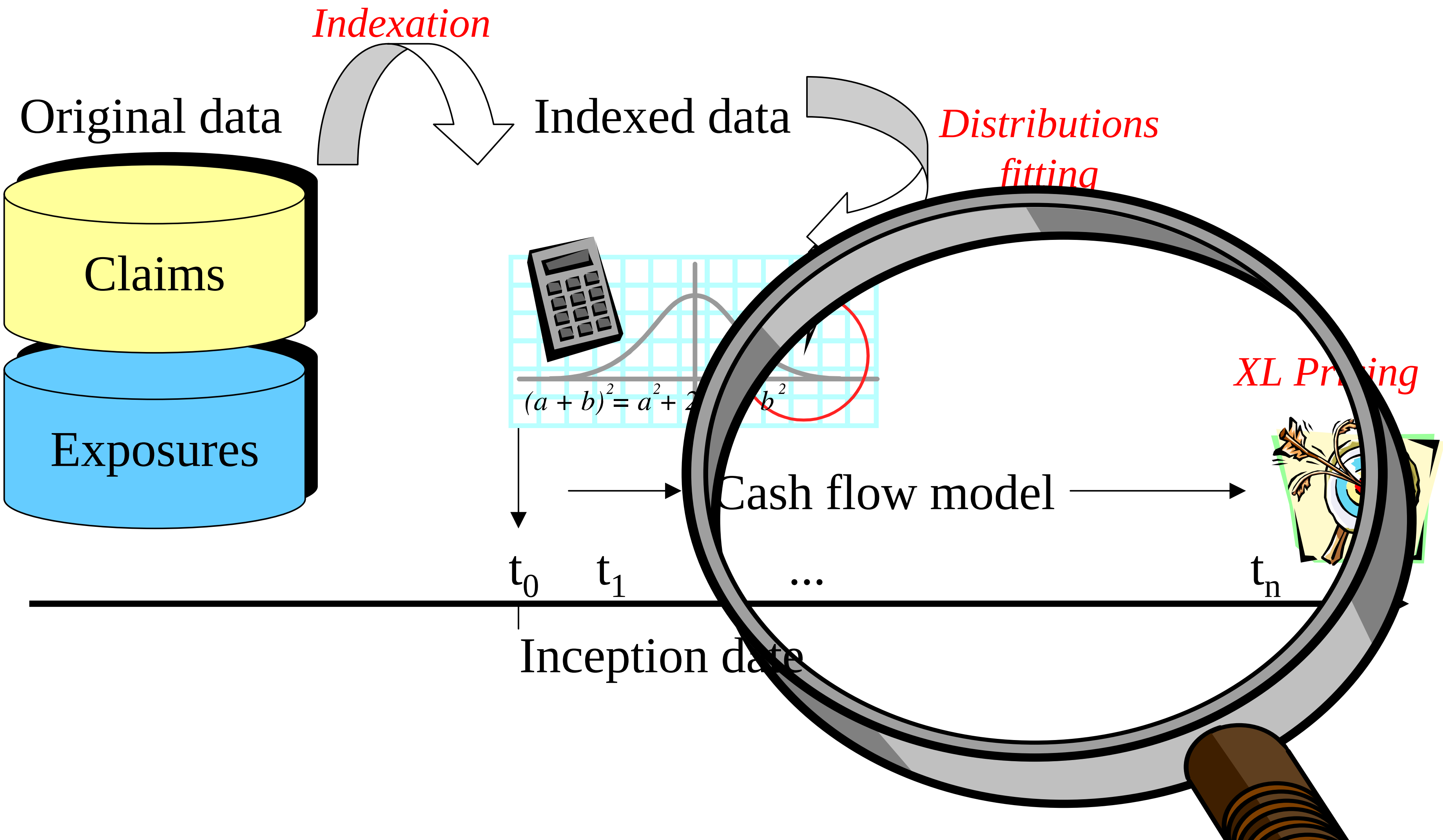
Changing the particularities

- **As a conclusion we can say that within this new methodology, we will study the real variables.**
- **Sensitivity analysis will be easy. Distributions won’t need to be reestimated.**

The problem ?

- **Loss modelling is requested :**
  - unobserved part of the treaty
  - specific clauses (reinstatements, aggregate deductibles, sliding scale premium, …)
- **Cash-flow modelling is requested :**
  - all cash flows do not occur at the same time
  - net present value

The proposed process



Steps

- ⌘ **Step 1 : As if statistic : transforming the past data to the pricing year.**
- ⌘ **Step 2 : parametric fit for the distribution of the observed random variables : X and N. Discretization of X.**
- ⌘ **Step 3 : projection in the future. Computation of expectations and cash flow model.**

Projection in the future

- Let X_i be the loss without future inflation.
- Let c(j) be the payment pattern. So we expect the loss to be paid in the future according to this pattern : $c(0)X_i, c(1)X_i, \ldots, c(n)X_i$.
- Future payments will be inflated : $X_i(j) = c(j)X_i \text{inf}(j)/\text{inf}(0)$.
- Future incurred losses (paid + outstanding) may also be modelled.

Clauses on individual claims

- **Stability clause : the priority and limit of the treaty are indexed in order that future inflation be shared between the cedent and the reinsurer : P and L become a function of j.**

Clauses on individual claims

- **Interests sharing clause : legal interests are shared on a pro rata basis between the cedent and the reinsurer. They are not counted as a part of the loss and thus they are not paid entirely by the reinsurer.**
- **Loss adjustment expenses clause : same principle : expenses are shared on a pro rata basis between the cedent and the reinsurer.**

Aggregate distributions

- **Aggregate payments, aggregate incurred losses (for the reinsurer) may be obtained (recursively) for each time j.**
- **This is an application of Panjer's algorithm for each j.**

Clauses on the aggregate losses

- **Annual aggregate deductible (Aad) / Annual aggregate limit (Aal) : $S^{clau}(j)=\min(Aal,\max(0,S(j)-Aad))$.**
- **It is then possible to compute the expected future payments as well as the evolution of the expected value of the future incurred loss.**

Cash-flows models

- ⌘We have to compare cash flows at different moments in the future.
- ⌘The underwriting decision must be profitable for the shareholder. We therefore analyse it with the shareholder's eyes.

Cash-flows models

- **Value of time : a Euro is more value now than tomorrow : discounting future cash flows at the risk free rate.**
- **Without risk 1 Euro in 10 years is worth $1/(1+r)^{10}$ Euro today if r denotes the risk free rate.**

Cash-flows models

- ⌘Value of risk : a safe Euro is more value than a risky Euro : discounting future cash flows at a risk-adapted rate.
- ⌘With risk 1 Euro in 10 years is worth $1/(1+k)^{10}$ today if k denotes some risk adapted rate ; $k > r$.

Cash-flows models

- ⌘Future cash flows are realizations of random variables.
- ⌘Future cash flows are just expectations. Risk of deviation is reflected in the risk adapted rate.

Cash-flows models

- Reinsurance is a risky business.
- We then estimate all future cash flows and discount them at a risk adapted rate.
- Risk adapted rate may e.g. be derived from the CAPM.
- One speaks of the cost of capital.

Cash-flows are

- Premium
- Brokerage
- Payments
- Variation of the technical reserve
- Investment income on the technical reserve
- Retrocession costs

Cash-flows are

- Administrative expenses
- Capital allocation and release
- Investment income on allocated capital
- Tax

Net present value

- **Discounting all the cash flows and adding them provides the net present value of the treaty.**
- **If NPV=0 then shareholder’s demands just fullfilled.**
- **If NPV > 0 then value creation for the shareholder.**
- **If NPV < 0 then shareholder’s value destroyed.**

Technico-financial premium

- **Uses only cash flows related to losses.**
- **See .pdf for an example.**

Commercial premium

- **Uses all the cash flows.**
- **See .pdf for an example.**

Random premium

- **For some treaties, the premium may be random.**
- **By random we mean that the premium depends on the observed losses.**
- **For example : profit participation.**

$$
\begin{aligned}
PP &= \alpha \max(\beta P^{init} - S, 0) \\
P^{init} &= E(S) + E(PP)
\end{aligned}
$$

Profit participation

- ⌘Behave premium (and brokerage and tax) all other cash flows remain the same.
- ⌘Now different cash flows of premium occur. The initial premium at inception date. PP later if any. Adjustments on the PP if necessary.

Profit participation

- Iteratively we look for P^{init} such that the NPV of the business with premium adjustments be 0.
- Premium adjustments may be contractually foreseen only after a few years.
- see .pdf for numerical example.

Conclusion

- ⌘ **Actuarial aspects : X,N, patterns, clauses, retrocession.**
- ⌘ **Economic aspects : inflation, superimposed inflation.**
- ⌘ **Financial aspects : investment income, cost of capital.**
- ⌘ **Commercial aspects : brokerage, administrative costs.**

Conclusion

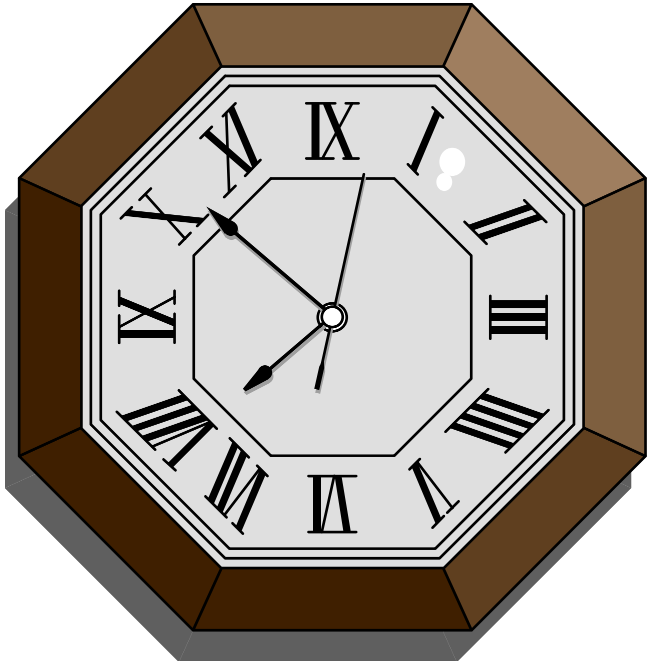
- ⌘Comprehensive methodology allowing for easy sensitivity analyses.
- ⌘Our main problem : step 1 : transformation of the data !

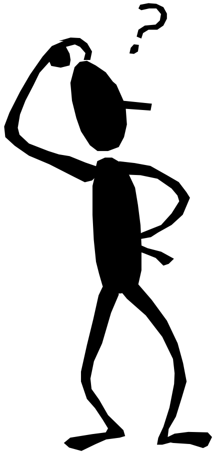
More results ?

- **Look at the BAB**
- **Belgian Actuarial Bulletin**
  - www.stat.ucl.ac.be\BAB

It’s time to stop ...

⌘ Thank you for your attention





Are there any further questions ?

- ⌘We would be pleased to answer