Is surplus reinsurance with table of lines optimal ?

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Agenda

- ✓ *Aim*
- ✓ **Surplus reinsurance**
- ✓ **Surplus reinsurance with table of lines**
- ✓ **Portfolio description**
- ✓ **Optimal reinsurance**
- ✓ **Conclusion**

Aim

Which reinsurance form is optimal for an insurance company :

Surplus reinsurance

OR

Surplus reinsurance with a table of lines

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Surplus reinsurance (1)

- ✓ Proportional reinsurance
- ✓ For each policy i a proportion α_i of the premium is ceded and a proportion α_i of the claim is paid back
- ✓ The proportion α_i is defined as follows (for Sum Insured > retention) :

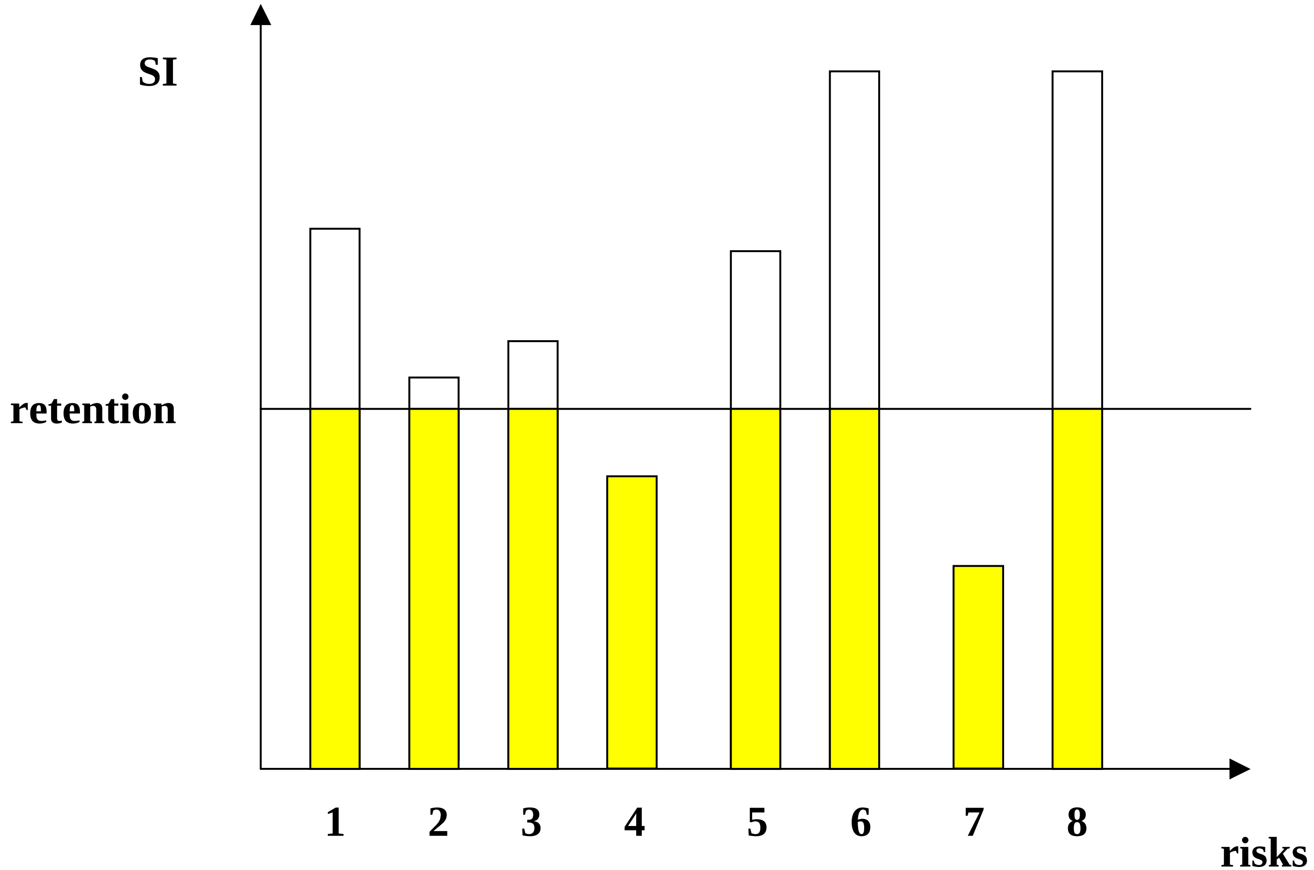
$$\frac{\text{SI} - \text{retention}}{\text{SI}}$$

Surplus reinsurance (2)

- ✓ For SI < retention : $\alpha_i = 0$
- ✓ The reinsured SI for each policy equals :

$$\max\left(0\,,\,SI - retention\right)$$

Surplus reinsurance (3)



Numerical example

	SI	Retention	Reinsured	α_i
Policy 1	20.000	10.000	10.000	50%
Policy 2	8.000	10.000	0	0%
Policy 3	50.000	10.000	40.000	80%
Policy 4	25.000	10.000	15.000	60%
Policy 5	12.000	10.000	2.000	17%

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Surplus reinsurance with table of lines (1)

- ✓ Multiple retentions instead of one retention for the entire portfolio
- ✓ Retention depending on the claim probability of the policy
- ✓ Result : an equal average loss in retention for the entire portfolio of the ceding company, independent of the individual claim probabilities

Example without table of lines

	Claim probability	Retention	Average loss
Class 1	10%	10.000	1.000
Class 2	20%	10.000	2.000
Class 3	30%	10.000	3.000

Example with table of lines

	Claim probability	Retention	Average loss
Class 1	10%	10.000	1.000
Class 2	20%	5.000	1.000
Class 3	30%	3.333	1.000

Surplus reinsurance with table of lines (2)

- ✓ In that way a homogeneous portfolio in retention is created for the ceding company
- ✓ This form of reinsurance is therefore believed to be better for the insurer than surplus reinsurance with one retention :

TRUE ??

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Portfolio description (1)

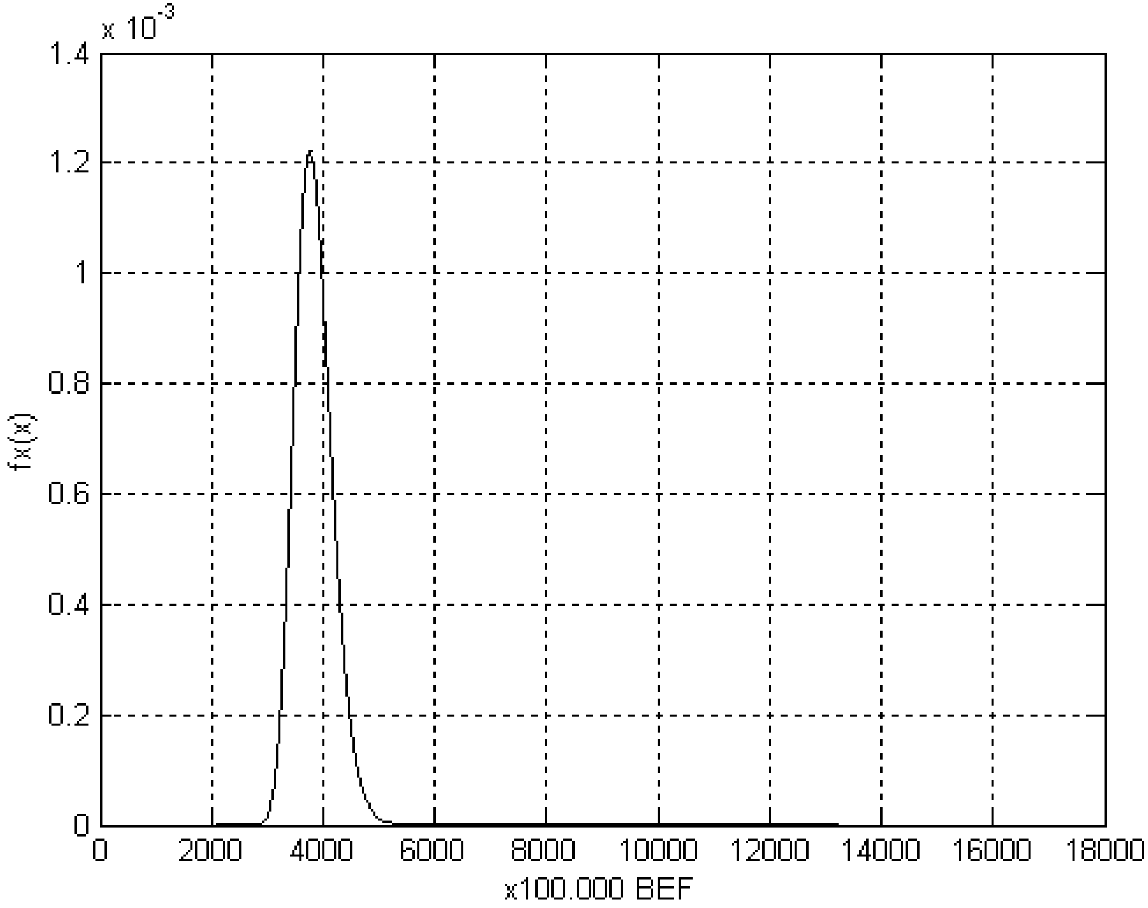
- ✓ **Fire portfolio with 27.500 policies, from a Belgian insurer**
- ✓ **4 classes of risk with for each class a different claim probability :**

Class 1	**4,7%**
Class 2	**3,2%**
Class 3	**4,4%**
Class 4	**2,8%**

Portfolio description (2)

- ✓ Next to the claim probabilities we have also the claim distribution for each policy, given there has been a claim (= the conditional claim distribution)
- ✓ With that we can calculate the (unconditional) claim distribution for each policy
- ✓ Calculation of the aggregate claim distribution of the entire portfolio through convolution

Aggregate claim amount distribution



Portfolio description (3)

We will apply different reinsurance programs on this portfolio :

- **no reinsurance**
- **surplus reinsurance with one retention**
- **surplus reinsurance with a table of lines**

in order to find out which is the optimal program for the insurer.

Reinsurance programs

- ✓ **Program 1 : no reinsurance**
- ✓ **Program 2 : surplus reinsurance with retention = 34.325.974 for all policies**
- ✓ **Program 3 : surplus reinsurance with table of lines such that $E(G_3) = E(G_2)$**

Program with table of lines

	Claim probability	Retention
Class 1	4,70%	29.574.468
Class 2	3,20%	43.437.500
Class 3	4,40%	31.590.909
Class 4	2,78%	50.000.000

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Optimal reinsurance

- ✓ How to determine which reinsurance program is better, which program is optimal for the insurer ?
- ✓ We use the risk-return model : ‘better’ means
  - more return for the same risk level
  - less risk for the same return level

➔ need for a definition of risk and return

Definition of return

- ✓ **Return for the ceding company :**

  original earned premium

  - the reinsurance premium

  - claims incurred in retention

- ✓ **Or :**

$$E(G) = E(c) - E(c_h) - E(S_h)$$

Definition of risk

✓ **We use different risk measures :**

- **standard deviation**
- **ruin probabilities**
- **VaR**
- **tailVaR or Conditional Tail Expectation (CTE)**
- **Wang Transform (WT)**

Standard deviation (1)

- ✓ Standard deviation of the aggregate claim amount distribution
- ✓ Commonly used measure
- ✓ Gives an idea of the spread of the loss amounts around the mean of the distribution
- ✓ Result : one retention means less risk than a table of lines given the same expected profit

Standard deviation (2)

	Standard deviation
Program 1 **(no reinsurance)**	**34.253.065**
Program 2 **(one retention)**	**32.466.991**
Program 3 **(table of lines)**	**32.518.788**

Ruin probabilities (1)

- ✓ We define a surplus process :

$$U_t = u + c.t - (S_1 + ... + S_t)$$

with - U_t the surplus of year t

- u the initial surplus

- c the earned premium per year

- S_i the yearly aggregate claim amount

- ✓ The finite time ruin probabilities are :

$$\psi(u,t) = P\left(\exists\, i \,\middle|\, U_i < 0,\, i = 1, 2, ..., t\right)$$

Ruin probabilities (2)

	$\psi(u,5)$
Program 1 **(no reinsurance)**	$2{,}92 \,.\, 10^{-9}$
Program 2 **(one retention)**	$2{,}03 \,.\, 10^{-18}$
Program 3 **(table of lines)**	$3{,}22 \,.\, 10^{-18}$

Value at Risk (1)

- ✓ Based on a percentile concept
- ✓ Only the frequency of the default, not the size
- ✓ VaR(α) = amount of money such that the loss will be less than that amount with a specified probability α :

$$\mathrm{VaR}(\alpha) = \mathrm{Min}\left\{ x \,\middle|\, F(x) \geq \alpha \right\}$$

Value at Risk (2)

	VaR(0,95) (* 100.000)
Program 1 (no reinsurance)	4.435
Program 2 (one retention)	4.358
Program 3 (table of lines)	4.360

TailVaR or CTE (1)

- ✓ Conditional Tail Expectation
- ✓ Takes also into account the (expected) size of the shortfall :

$$\mathrm{CTE}(\alpha) = \mathrm{VaR}(\alpha)$$

$$+ \frac{P\left(X \geq \mathrm{VaR}(\alpha)\right)}{1-\alpha} \,.\, E\left(X - \mathrm{VaR}(\alpha) \,\middle|\, X > \mathrm{VaR}(\alpha)\right)$$

TailVaR or CTE (2)

	<u>CTE(0,95)</u> (*100.000)
Program 1 **(no reinsurance)**	**4.648**
Program 2 **(one retention)**	**4.792**
Program 3 **(table of lines)**	**4.796**

Wang Transform (1)

- ✓ **Takes into account the entire distribution function (and not only above the VaR)**
- ✓ **Calculation :**

1) **For a preselected security level α, let $\lambda = \Phi^{-1}(\alpha)$.**
2) **Apply the Wang Transform : $F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$.**
3) **Set the capital requirement at the expected value under F* : $WT(\alpha) = E^*(X)$.**

Wang Transfrom (2)

	WT(0,95) (*100.000)
Program 1 (no reinsurance)	4.538
Program 2 (one retention)	4.381
Program 3 (table of lines)	4.383

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Conclusion

- ✓ All risk measures indicate that surplus reinsurance with a table of lines is LESS optimal than surplus reinsurance with one retention (also when changing parameters such as loadings, retention, security level, ...)
- ✓ BUT the results are not very convincing (the difference is small)
- ✓ At least we have learned that it is not evident that a table of lines is better